Mathematics 120 Review for Midterm I
October, 2005

Definitions you should know:

1. Group, subgroup, normal subgroup.
2. Ring.
3. Isomorphism, homomorphism of groups or rings.
4. Abelian group
5. Equivalence relation.
6. Cosets of a subgroup.
7. The order of an element in a group (finite or infinite order).
8. The Euler φ function.

Theorems you should be able to prove:

1. If p is a prime integer, and p divides a product ab, then p divides a or p divides b.
2. The quotient by a normal subgroup is a group.
3. The kernel of a homomorphism is a normal subgroup. The homomorphism is injective if, and only if, the kernel is trivial (kernel = $\{e\}$).
4. Lagrange's theorem.
5. If a is relatively prime to n, then $a^{\varphi(n)} \equiv 1 \mod n$. (Euler's theorem).
6. A group of prime order is cyclic.
7. The basic homomorphism theorem.
8. The correspondence theorem.
9. The diamond isomorphism theorem.
10. The nameless isomorphism theorem (Propostion 2.7.13.)
11. If a is an element of order n in a group, then $\langle a \rangle \cong \mathbb{Z}_n$. (Use the basic homomorphism theorem, employing a surjective from $\mathbb{Z}$ to $\langle a \rangle \cong \mathbb{Z}_n$.
12. The classification of cyclic groups.
13. The classification of subgroups of $\mathbb{Z}$ and of $\mathbb{Z}_n$.
14. If a and b are relatively prime integers, each greater than 1, then $\mathbb{Z}_{ab} \cong \mathbb{Z}_a \times \mathbb{Z}_b$ as rings. Conclude that $\varphi(ab) = \varphi(a)\varphi(b)$, where φ denotes the Euler function.
15. The set of all 2-cycles generate S_n. The set of 2-cycles $\{(1,2),(2,3),\ldots,(n-1,n)\}$ generate S_n.

Be able to provide examples of:

1. A non-abelian group.
2. A non-normal subgroup , a normal subgroup.
3. A homomorphism which is neither injective nor trivial ($\varphi(G) \neq \{e\}$.)
4. An example of an application of each of the isomorphism theorems listed above.

About 3/4 of the exam will be things you can do by preparing thoroughly from this review sheet. The remainder will be one or more exercises from the homework, or similar to the homework.