Some terminology and notation regarding functions

A function f is a rule which assigns to each element of some set D (called the domain of the function) an element of another set E (called the codomain of the function). All this information is summarized by the notation $f : D \to E$.

The sets D and E can consist of objects of any sort. For example, we could define a function A which assigns to each person in our class the age, in years, of that person. The domain of the function would be the set of persons in our class. Some choice is possible for the codomain; a suitable codomain would be the set $\mathbb{N}$ of natural numbers, $\mathbb{N} = \{1, 2, 3, \dots\}$. Thus we write:

$$A : \{\text{persons in our class}\} \to \mathbb{N}.$$

Notice that domain and codomain of this function consist of objects of entirely different types; the domain is a set of people and the codomain is a set of integer numbers.

The range of a function $f : D \to E$ is the set of values actually taken by the function,

$$\text{range}(f) = \{f(x) : x \in D\}.$$

It is a subset of E. The age function described above is a perfectly good, well defined function, but we don't know its range until we actually ask all the persons in our class for their age. The range of A might be

$$\text{range}(A) = \{18, 19, 20, 30\},$$

if our class has members of ages 18, 19, 20, and 30, and no members of any other age.

Example: Let $f(x) = x^{14} - 2x^5 + 3x^2 - 5$, a polynomial function. A suitable domain for this function is the set $\mathbb{R}$ of all real numbers. The function takes real number values, so a suitable codomain is $\mathbb{R}$ as well. We write

$$f : \mathbb{R} \to \mathbb{R}.$$

It's not obvious what the range of the function is. You could try to find the range, approximately, by asking a computer program or graphing calculator to graph the function.

The reason why it is useful to have a notion of codomain, distinct from the notion of range is that we would like to be able to say what type of values a function can take, without actually determining the exact range. In the last example, the function f is a real valued function of a real variable; it accepts any real number input (its domain is $\mathbb{R}$) and yields a real number output (its codomain is $\mathbb{R}$). But we don't know exactly which real number outputs are obtained.