Mathematics 25 Review for Midterm I
September, 2009

No calculators will be needed or allowed on the exam.

Definitions you should be able to state. *You should learn the definition verbatim from the text or from class notes. If instead you try to express the idea in your own words, it is extremely likely that you will get it wrong, either because you don't understand the idea well enough yet, or because you are unable to precisely express the idea.*

1. $\lim_{x \to a} f(x) = L$. (page 88 of text)
2. A function $f(x)$ is continuous at $x = a$. (page 119 of text)
3. The definition of the derivative of a function $f(x)$ at a point $x = a$. (page 146 of text, or formula (5) on page 147)
4. A function $f(x)$ is strictly increasing on an interval J. (definition given in class)

You should be familiar with terminology and notation given in class, for example the notion of the codomain of a function, or the notation $f : D \to E$.

Theorems you should be able to state. *Same warning applies as for definitions.*

1. The intermediate value theorem, page 126.

Skills you need to have:

1. Find the derivative of a given function $f(x)$ at $x = a$ directly from the definition of derivative.
2. Use the derivative to find the tangent line to the graph of a function at a certain point $(a, f(a)$.
3. Use the derivative of the position function of an object to find the instantaneous velocity of the object. Distinguish between average velocity and instantaneous velocity.
4. Use the limit laws to calculate limits (putting in all the tedious steps).

Sample questions:

1. Find the derivative of the function $f(x) = \sqrt{x}$ directly from the definition of derivative. In doing the caclulation, you will have to evaluate a limit. Carefully use laws for limits to evaluate the limit, including all steps.
2. The derivative of $f(x) = x^3 + 2x^2$ is $f'(x) = 3x^2 + 4x$. Using this, find the tangent line to the graph of the function at $(2, 16)$.
3. An object moves along a straight track with position described by the function $s(t) = t^3 + 2t^2$, where position and time are measured in meters and seconds, respectively. Find the average velocity of the object during the time interval from $t = 2$ to $t = 5/2$. Find the instantaneous velocity at time $t = 2$.

4. Explain in detail why the function

$$f(x) = \sqrt[3]{\frac{x^5 + 1}{(x - 1)(x + 2)}}$$

is continuous except at $x = 1$ and $x = -2$.

5. Any homework exercise.