22M027: Introduction to Linear Algebra

Chapter 1 Review

- Linear equations.
- Geometrical interpretation.
- Number of solutions of a linear system.
- Solving linear systems by row reductions.
- Row-echelon form and reduced row-echelon form of a matrix.
- Matrix operations (addition, scalar mutiplication, multiplication including block multiplication) and their properties; matrix transpose.
- Matrix inverse: elementary matrices, finding matrix inverse, using inverses in solving linear system.
- Invertible Matrix Theorem.
- Scalar, diagonal, triangular and symmetric matrices and their properties.

1. Mark each statement either True or False. Justify your answer.

   (a) If a matrix is reduced to reduced row-echelon form by two different sequences of elemenary row operations, the resulting matrices will be different.

   (b) Every matrix is row equivalent to a unique matrix in row-echelon form.

   (c) If three lines in the xy-plane are sides of a triangle, then the system formed from their equations has three solutions, one corresponding to each vertex.

   (d) A linear system of three equations in five unknowns must be consistent.

   (e) A linear system of five equations in three unknowns cannot be consistent.

   (f) If a system of linear equations has two different solutions, then it has infinitely many solutions.

   (g) If a system $Ax = b$ has more than one solutions, then so does the system $Ax = 0$.

   (h) $Ax = 0$ implies $x = 0$.

   (i) If matrices A and B are row equivalent then they have the same reduced echelon form.

(j) If A is an $m \times n$ matrix and if the equation $Ax = b$ has a solution for every b in $\mathbb{R}^m$, then A has m pivot columns.

(k) If an $m \times n$ matrix A has a pivot position in every row, then the equation $Ax = b$ has a solution for every b in $\mathbb{R}^m$.

(l) If an $n \times n$ matrix A has n pivot positions, then the reduced echelon form of A is the $n \times n$ identity matrix.

(m) Every square matrix can be expressed as a product of elementary matrices.

(n) The product of two elementary matrices is an elementary matrix.

(o) If A is invertible and $AB = O$, then it must be true that $B = O$.

2. Let A and B be $n \times n$ matrices. Indicate whether the following statements are always true or sometimes false:

   (a) $(AB)^2 = A^2B^2$.

   (b) $(AB^{-1})(BA^{-1}) = I_n$.

   (c) $(A + B)^2 = A^2 + 2AB + B^2$.

3. Find a formula for the product of two diagonal matrices.

4. Use **block product** to find the product AB of the following matrices

$$A := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \qquad B := \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$