Combinations of Sets

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A combination is an unordered collection of objects from a given set.
An r-combination of an n-set is an r-subset of S. The number of r-combinations of an n-set is denoted by $\binom{n}{r}$ or $C(n, r)$.

Theorem 1 The number of r-combinations of an n-set equals

$$\binom{n}{r} = \frac{P(n, r)}{r!} = \frac{n!}{(n-r)!r!}.$$

Theorem 2 The any integers n, r suct that $n \geq r \geq 0$,

$$\binom{n}{r} = \binom{n}{n-r}.$$

Theorem 3 The number of subsets of an n-set equals

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n.$$

1. Twenty-five points are chosen in the plane so that no three of them are collinear. How many straight lines do they determine? How many triangels do they determine?

2. How many 8-letter words can be constructed from 26 letters of the alphabets if each word contains 3, 4, or 5 vowels? It is understood that there is no restriction on the number of times a letter can be used in a word.

3. How many shortest paths are there from one corner of a 9×8 grid to the opposite corner?