Two Counting Principles

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Addition Principle. Let $S_1, S_2, \ldots S_k$ be pairwise disjoint subsets of a finite set S. If $S = S_1 \cup S_2 \cup \cdots \cup S_k$, then

$$|S| = |S_1| + |S_2| + \cdots + |S_k|.$$

Multiplication Principle. Let $S_1, S_2, \ldots S_k$ be finite sets and let the set S be $S = S_1 \times S_2 \times \cdots \times S_k$, then

$$|S| = |S_1| \times |S_2| \times \cdots \times |S_k|.$$

1. Determine the number of positive integers that are factors of the number $3^2 \cdot 5^3 \cdot 7^2 \cdot 11$.
2. How many two-digit numbers have distinct and nonzero digits?
3. How many odd numbers between 1000 and 9999 have distinct digits?
4. In how many ways to make a basket of fruit from 6 oranges, 7 apples, and 8 bananas so that the basket contains at least two apples and one banana?
5. How many integers between 0 and 10,000 have exactly one digit equal to 5?
6. How many distinct 5-digit numerals can be constructed out of the digits $1, 1, 1, 6, 8$?
7. Determine the largest power of 10 that is a factor of 50!

General Ideas about Counting:

1. Count the number of ordered arrangements or ordered selections of objects
   (a) without repetition,
   (b) with repetition allowed.
2. Count the number of unordered arrangements or unordered selections of objects
   (a) without repetition,
   (b) with repetition allowed.

A multiset M is a collection whose members need not be distinct. For instance, the collection

$$M = \{a, a, b, b, c, d, d, d, f, g, g, g\}$$

is a multiset; and sometimes it is convenient to write

$$M = \{2a, 2b, 1c, 3d, 1f, 3g\}$$

A multiset M over a set S can be viewed as a function $v : S \to N$ from S to the set N of nonnegative integers.