22M150: Introduction to Discrete Mathematics.

Homework 11 due 11/19/08.

Use ordinary or exponential generating functions to solve the following problems:

1. Find the number of 5-combinations of the multiset $\{4 \cdot a, 4 \cdot b, 4 \cdot c\}$.

2. How many ways are there to distribute 3000 identical envelopes, packed in blocks of 25, into 4 boxes so that each box contains between 150 and 1000 envelopes.

3. Find the number of nonnegative integer solutions for the equation

$$y_1 + 2y_2 + 2y_3 = n.$$

4. Find the closed form for the (ordinary) generating function of the sequence $a_i = i^2$.

5. Solve the recurrence relation $a_n = 2a_{n-1} + 2^n, a_0 = 1$.

6. Evaluate the sum $\sum_{i=0}^{k} \binom{k-i}{i} 2^i$.

7. Find the closed form for the exponential generating function of the sequence $a_i = \frac{1}{i+1}$.

8. How many ways are there to make an r-arrangement of pennies, nickles, dimes, and quarters with at least one penny and an odd number of quarters? (Coins of the same denomination are identical).