Binomial Identities

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(1)
$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1} \qquad (1 \le k \le n-1)$$

(2)
$$\binom{n}{k} = \binom{n}{n-k} \qquad (0 \le k \le n)$$

(3)
$$k\binom{n}{k} = n\binom{n-1}{k-1} \qquad (1 \le n)$$

(4)
$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n \qquad (n \ge 0)$$

(5)
$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n \binom{n}{n} = 0 \qquad (n \ge 1)$$

(6)
$$\binom{n}{0} + \binom{n}{2} + \cdots = \binom{n}{1} + \binom{n}{3} + \cdots (= 2^{n-1}) \qquad (n \ge 1)$$

(7)
$$1\binom{n}{1} + 2\binom{n}{2} + \cdots + n\binom{n}{n} = n2^{n-1} \qquad (n \ge 1)$$

(8)
$$1^2\binom{n}{1} + 2^2\binom{n}{2} + \cdots + n^2\binom{n}{n} = n(n+1)2^{n-2} \qquad (n \ge 1)$$

(9)
$$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n} \qquad (n \ge 0)$$

(10)
$$\binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} + \cdots + \binom{n+k}{k} = \binom{n+k+1}{k}$$

(11)
$$\binom{0}{k} + \binom{1}{k} + \binom{2}{k} + \cdots + \binom{n}{k} = \binom{n+1}{k+1} \qquad (1 \le k \le n)$$