Permutation of Multisets

September 16, 2008

An r-permutation of M is a linearly ordered arrangements of r objects of M. If $|M| = n$, then an n-permutation of M objects is called a **permutation of** M.

Theorem 1 Let $M = \{\infty \cdot x_1, \infty \cdot x_2, \ldots, \infty \cdot x_k\}$ be a multiset of k different types where each type has infinitely many elements. Then the number of r-permutations of the multiset M equals

$$k^r.$$

Theorem 2 Let $M = \{n_1 \cdot x_1, n_2 \cdot x_2, \ldots, n_k \cdot x_k\}$ be a multiset of k types with repetition numbers $n_1, n_2, \ldots, n_k$ respectively, and $n = n_1 + n_2 + \cdots + n_k$. Then the number of permutations of the multiset M equals

$$\frac{n!}{n_1! n_2! \cdots n_k!}.$$

Corollary 3 The number of $0 - 1$ words of length n with exactly r ones and $n - r$ zeros is equal to

$$\frac{n!}{(n-r)!r!} = \binom{n}{r}.$$

Theorem 4 Given n rooks of k colors with n_1 rooks of the first color, n_2 rooks of the second color, ..., and n_k rooks of the kth color. The number of ways to arrange these rooks on an n-by-n board so that no two rooks can attack another equals

$$n! \times \frac{n!}{n_1! n_2! \cdots n_k!} = \frac{(n!)^2}{n_1! n_2! \cdots n_k!}.$$

1. What is the number of ternary numerals with at most 4 digits?
2. Find the number of permutations of the letters in the word "MISSISSIPPI".
3. In how many ways can 8 identical rooks be placed on an 8-by-8 chessboard so that no two rooks can attak one another?
4. In how many ways can 8 rooks of different color be placed on an 8-by-8 chessboard so that no two rooks can attak one another?
5. How about possibilities are there for one red rook, 3 blue rooks , and 4 yellow?
6. In how many ways can 8 identical non-attacking rooks be placed on a 12-by-12 chessboard?
7. Find the number of 8-permutations of the multiset $M = \{3a, 2b, 4c\}$.