Permutation of Sets

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An r-permutation of n objects is a linearly ordered selection of r objects from a set of n objects. The number of r-permutations of n objects is denoted by $P(n, r)$.
An n-permutation of n objects is called a **permutation of n objects**.
Theorem 1 The number of r-permutations of an n-set equals

$$P(n, r) = n(n-1)\cdots(n-r+1) = \frac{n!}{(n-r)!}.$$

Corollary 2 The number of permutations of an n-set is $P(n, n) = n!$.
A **circular r-permutation** of a set S is an ordered selection of r objects of S arranged as a circle; there is no the beginning object and the ending object.
Theorem 3 The number of circular r-permutations of an n-set equals

$$\frac{P(n, r)}{r} = \frac{n!}{(n-r)!r}.$$

Corollary 4 The number of circular permutations of an n-set is equal to $(n-1)!$

1. Find the number of positions in the "15 puzzle".
2. Find the number of ways to put the numbers $1, 2, \ldots 8$ into the squares of 6-by-6 grid so that each square contains at most one number.
3. Find the number of ways to arrange the 26 letters of the alphabet so that no two of the vowels a, e, i, o, and u occur next to each other?
4. Find the number of 7-digit numbers such that all digits are nonzero, distinct, and the digits 8 and 9 do not appear next to each other.
5. Twelve people, including two who do no wish to sit next to each other, are to be seated at a round table. How many circular seating plans can be made?
6. How many different patterns of necklaces with 18 beads can be made out of 25 available beads of the same size but in different colors?
7. In how many ways can six men and six ladies be seated at a round table if the men and ladies are to sit in alternative seats?