Pigeonhole Principle (Simple form)

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If you put $n+1$ pigeons in n pigeonholes then at least one hole would have more than one pigeon.

1. Among 13 people there are two who have their birthdays in the same month.

2. You have 12 pairs of black socks and 12 pairs of brown socks. You take, with eyes closed, some number of socks. What is the least number of socks you have to take in order to ensure that we get at least 2 socks of the same color?

3. There are n married couples. How many of the $2n$ people must be selected in order to guarantee that one has selected a married couple?

4. There are 50 people in the room. Some of them are acquainted with each other, some not. Prove that there are two persons in the room who have equal numbers of acquaintances.

5. We are given m arbitrary natural numbers $a_1, a_2, \ldots, a_m$. Prove that the sum of some consecutive numbers in the sequence is divisible by m.

6. A chess master who has 11 weeks to prepare for a tournament decides to play at least one game every day but, in order not to tire himself, he decides not to play more than 12 games during any calendar week. Show that there exists a succession of consecutive days during which the chess master will have played exactly 21 games.

7. Given 101 integers from $1, 2, \ldots, 200$, there are at least two integers such that one of them is divisible by other.

8. (a) 5 points are positioned inside of the equilateral triangle of side 1. Prove that there are two of them at the distance at most 0.5 from each other.

   (b) 10 points are positioned inside of the equilateral triangle of side 1. Prove that there are two of them at the distance at most $\frac{1}{3}$ from each other.