Review I

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If you put $n+1$ pigeons in n pigeonholes then at least one hole would have more than one pigeon.
If $n(r-1)+1$ objects are put into n boxes, then at least one of the boxes contains r or more of the objects.
If the average of n nonnegative integers $a_1, a_2, \ldots a_n$ is greater than $r-1$, i.e.,

$$\frac{a_1 + a_2 + \cdots + a_n}{n} > r - 1,$$

then at least one of the integers is greater than or equal to r.
The number of r-permutations of an n-set equals

$$P(n,r) = n(n-1)\cdots(n-r+1) = \frac{n!}{(n-r)!}.$$

The number of permutations of an n-set is $P(n,n) = n!$.
The number of circular r-permutations of an n-set equals

$$\frac{P(n,r)}{r} = \frac{n!}{(n-r)!r}.$$

The number of circular permutations of an n-set is equal to $(n-1)!$
The number of r-combinations of an n-set equals

$$\binom{n}{r} = \frac{P(n,r)}{r!} = \frac{n!}{(n-r)!r!}.$$

The number of r-permutations of the multiset $\{\infty \cdot x_1, \infty \cdot x_2, \ldots, \infty \cdot x_k\}$ equals k^r.
The number of permutations of the multiset $\{n_1 \cdot x_1, n_2 \cdot x_2, \ldots, n_k \cdot x_k\}$ equals

$$\frac{n!}{n_1! n_2! \cdots n_k!}, \qquad \text{where } n = n_1 + n_2 + \cdots + n_k$$

Then the number of r-combinations of the multiset $\{\infty \cdot a_1, \infty \cdot a_2, \ldots, \infty \cdot a_k\}$ (the number of r-combinations with repetition allowed) equals $\binom{k+r-1}{r} = \binom{k+r-1}{k-1}$.
The number of nonnegative integer solutions for the equation $x_1 + x_2 + \cdots + x_k = r$ equals $\binom{k+r-1}{r} = \binom{k+r-1}{k-1}$.
The number of positive integer solutions for the equation $x_1 + x_2 + \cdots + x_k = r$ equals $\binom{r-1}{k-1}$.
The number of ways to place r identical balls into k distinct boxes equals $\binom{k+r-1}{r} = \binom{k+r-1}{k-1}$.
The number of ways to place r identical balls into k distinct boxes such that no box remains empty equals $\binom{r-1}{k-1}$.

Algorithm for generating the permutations of $\{1, 2, \ldots, n-1, n\}$:
Begin with $\overleftarrow{1}\,\overleftarrow{2} \cdots \overleftarrow{n}$.
While there exists a mobile integer, do
(1) Find the largest mobile integer m
(2) Switch m and the adjacent integer its arrow points to.
(30 Switch thew direction of all the arrows above integers p with $p > m$.

Algorithm 1 for construction of a permutation from its inversion sequence $(a_1, a_2, \ldots, a_n)$:
(n) Write down n.
...
(n-k) Insert $n-k$ to the right of the a_{n-k}th existing number
...

Algorithm 2 for construction of a permutation from its inversion sequence $(a_1, a_2, \ldots, a_n)$:
(0) Mark down n empty spaces.
For $k = 1$ till n
Put k into the $a_k + 1$st empty space from the left.

Algorithm for generating combinations of $\{x_{n-1}, x_{n-2}, \ldots, x_1, x_0\}$:
Begin with $a_{n-1}a_{n-2} \cdots a_1 a_0 = 00 \ldots 00$.
While $a_{n-1}a_{n-2} \cdots a_1 a_0 \neq 11 \ldots 11$, do
(1) Find the smallest integer j such that $a_j = 0$.
(2) Replace a_j by 1 and each of $a_{j-1}, \ldots, a_1, a_0$ by 0.
The algorithm stops when $a_{n-1}a_{n-2} \cdots a_1 a_0 = 11 \ldots 11$.

Algorithm for generating reflected Gray codes of order n:
Begin with $a_{n-1}a_{n-2} \cdots a_1 a_0 = 00 \ldots 00$.
While $a_{n-1}a_{n-2} \cdots a_1 a_0 \neq 10 \ldots 00$, do
(1) If $a_{n-1} + a_{n-2} + \cdots + a_1 + a_0 =$ even, then change a_0 (from 0 to 1 or 1 to 0).
(2) If $a_{n-1} + a_{n-2} + \cdots + a_1 + a_0 =$ odd, find the smallest j such that $a_j = 1$ and change a_{j+1} (from 0 to 1 or 1 to 0).

Algorithm for generating r-combinations of $S = \{1, 2, \ldots, n-1, n\}$:
Begin with $12 \cdots r$.
While $a_1 a_2 \cdots a_r \neq (n-r+1) \cdots (n-1)n$, do
(1) Find the largest integer k such that $a_k < n$ and $a_k + 1$ is not in the $a_1 a_2 \cdots a_r$.
(2) Replace $a_1 a_2 \cdots a_r$ with

$$a_1 a_2 \cdots a_{k-1}(a_k + 1)(a_k + 2) \cdots (a_k + r - k + 1).$$

Algorithm for a linear extension of an n-poset:
Step 1. Choose a minimal element x_1 from X (with respect to the ordering $\leq$).
Step 2. Delete x_1 from X; choose a minimal element x_2 from X.
Step 3. Delete x_2 from X and choose a minimal element x_3 from X.
...
Step n. Delete x_{n-1} from X and choose the only element x_n in X.