

## 16. The Subspace Topology.

Defn: Let  $(X, \mathcal{T})$  be a topological space,  $Y \subset X$ . Then the **subspace topology** on  $Y$  is the set

$$\mathcal{T}_Y = \{U \cap Y \mid U \in \mathcal{T}\}$$

$(Y, \mathcal{T}_Y)$  is a **subspace** of  $X$ .

Lemma 16.1: If  $\mathcal{B}$  is a basis for the topology of  $X$ , then the set

$$\mathcal{B}_Y = \{B \cap Y \mid B \in \mathcal{B}\}$$

is a basis for the subspace topology on  $Y$ .

Lemma 16.2: Let  $Y$  be a subspace of  $X$ . If  $U$  is open in  $Y$  and  $Y$  is open in  $X$ , then  $U$  is open in  $X$ .

Lemma 16.3: If  $A_j$  is a subspace of  $X_j$ ,  $j = 1, 2$ , then the product topology on  $A_1 \times A_2$  is the same as the topology  $A_1 \times A_2$  inherits as a subspace of  $X_1 \times X_2$ .

Note: Suppose  $Y \subset X$  where  $X$  is an ordered set with the order topology. The order topology on  $Y$  need not be the same as the subspace topology on  $Y$

Ex 1:  $(0, 1) \cup \{5\}$

Defn: Suppose  $Y \subset X$  where  $X$  is an ordered set.  $Y$  is **convex** if for all  $a, b \in Y$  such that  $a < b$ , then  $(a, b) \subset Y$

Ex. 1:  $(1, 2) \cup (3, 4) \subset R$ .

Ex. 2:  $(1, 2) \cup (3, 4) \subset (1, 2) \cup (3, 9)$ .

Lemma 16.4: Let  $X$  is an ordered set with the order topology. Let  $Y$  be a convex subset of  $X$ . Then the order topology on  $Y$  is the same as the subspace topology on  $Y$ .

HW p91: 1, 3 (prove your answer), 8