

Math 150 Exam 2  
 November 3, 2006

[10] 1a.) What is the coefficient of  $x^3y^2z^5$  in the expansion of  $(2x+y-z)^{10}$  : \_\_\_\_\_

[6] 1b.) What is the coefficient of  $x^3y^2z^4$  in the expansion of  $(2x+y-z)^{10}$  : \_\_\_\_\_

[84] Choose 4 from the following 5 problems. Circle your choices: A B C D E  
 You may do all 5 problems in which case your unchosen problem can replace your lowest problem at 4/5 the value. Note you must fully explain your answers.

A.) Use Newtons binomial theorem to estimate  $\sqrt{5}$  (expand to at least 4 terms).

B.) Find the number of integers between 1 and 10,000 inclusive that are not divisible by 4, 6, 10.

C.) What is the number of ways to place ten nonattacking rooks on the 10-by-10 board with forbidden positions as shown?

X	X								
	X								
			X	X					

D.) Let  $R_n$  denote the number of permutations of  $X_n = \{1, 2, \dots, n\}$ ,  $n \geq 3$  in which neither the pattern 12 nor the pattern 23 occurs (note there are only 2 restrictions, for example, the pattern 34 may or may not occur). Determine a formula for  $R_n$  and prove your formula is correct.

E.) Consider the partially ordered set  $(\mathcal{P}(X_2), \subset)$  of subsets of  $\{1, 2\}$  partially ordered by containment. Let a function  $f$  in  $\mathcal{F}(\mathcal{P}(X_2))$  be defined by

$$f(A, B) = \begin{cases} 2 & \text{if } A = B \\ 3 & \text{if } A \subset B, A \neq B \\ 0 & \text{otherwise} \end{cases}$$

Find the following:

$$f^{-1}(\emptyset, \emptyset) = \underline{\hspace{2cm}} \quad f^{-1}(\emptyset, \{1\}) = \underline{\hspace{2cm}} \quad f^{-1}(\emptyset, \{2\}) = \underline{\hspace{2cm}}$$

$$f^{-1}(\emptyset, \{1, 2\}) = \underline{\hspace{2cm}} \quad (f * f)(\emptyset, \{1\}) = \underline{\hspace{2cm}}$$