Increasing/Decreasing Test:
If $f'(x) > 0$ for all $x \in (a, b)$, then f is increasing on (a, b)

If $f'(x) < 0$ for all $x \in (a, b)$, then f is decreasing on (a, b)

First derivative test:
Suppose c is a critical number of a continuous function f, then

Defn: f is **concave down** if the graph of f lies below the tangent lines to f.

Defn: f is **concave up** if the graph of f lies above the tangent lines to f.

Concavity Test:
If $f''(x) > 0$ for all $x \in (a, b)$, then f is concave upward on (a, b).

If $f''(x) < 0$ for all $x \in (a, b)$, then f is concave down on (a, b).

Defn: The point (x_0, y_0) is an **inflection point** if f is continuous at x_0 and if the concavity changes at x_0

Second derivative test: If f'' continuous at c, then
If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

If $f'(c) = 0$ and $f''(c) = 0$, second derivative test gives no info.

Converses are not true:

Increasing/Decreasing Test

If $f'(x) > 0$ for all $x \in (a, b)$, then f is increasing on (a, b)

f increasing on (a, b) does not imply $f'(x) > 0$ for all $x \in (a, b)$.

Ex:

If $f'(x) < 0$ for all $x \in (a, b)$, then f is decreasing on (a, b)

f decreasing on (a, b) does not imply $f'(x) < 0$ for all $x \in (a, b)$.

Ex:

Concavity Test:
If $f''(x) > 0$ for all $x \in (a, b)$, then f is concave upward on (a, b).

f concave upward on (a, b) does not imply $f''(x) > 0$ for all $x \in (a, b)$.

Ex:

If $f''(x) < 0$ for all $x \in (a, b)$, then f is concave down on (a, b).

f concave downward on (a, b) does not imply $f''(x) < 0$ for all $x \in (a, b)$.

Ex: