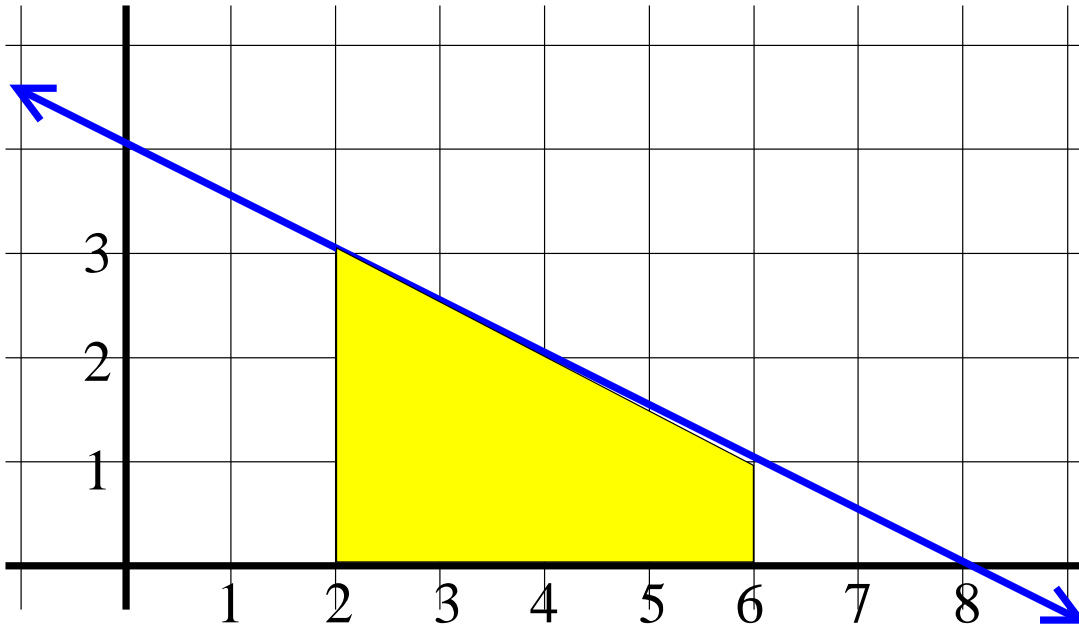


Section 5.2

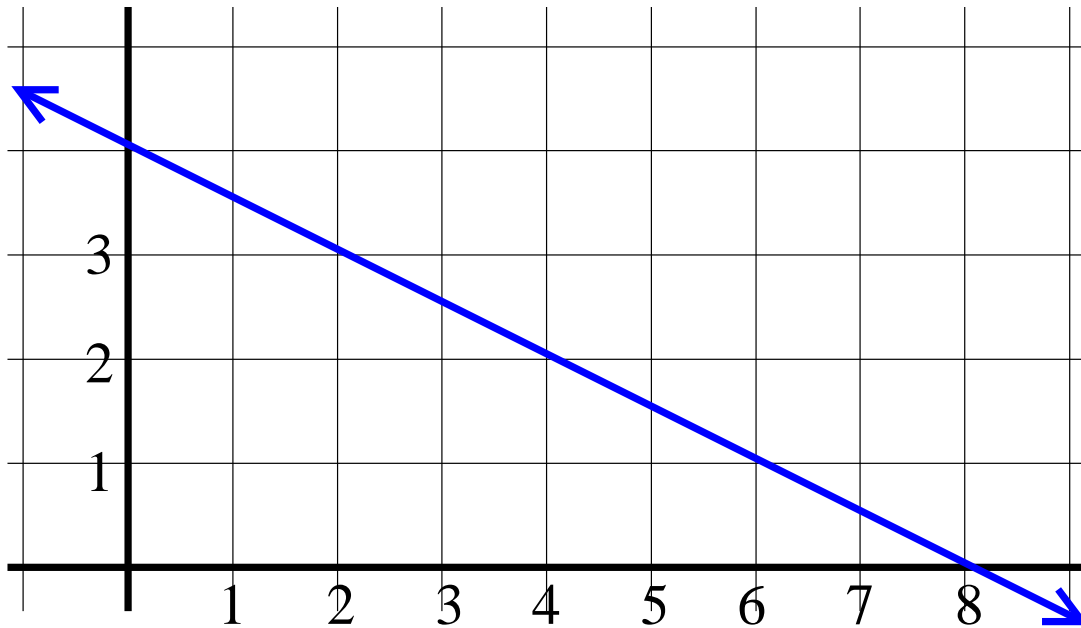
Find the area under the curve $f(x) = -\frac{1}{2}x + 4$, above the x -axis and between $x = 2$ and $x = 6$.



Method 1: In this case our function is very simple, so we can determine the area without calculus:

Method 2: Estimate using rectangles.

Inscribed rectangles with $\Delta x = 1$:

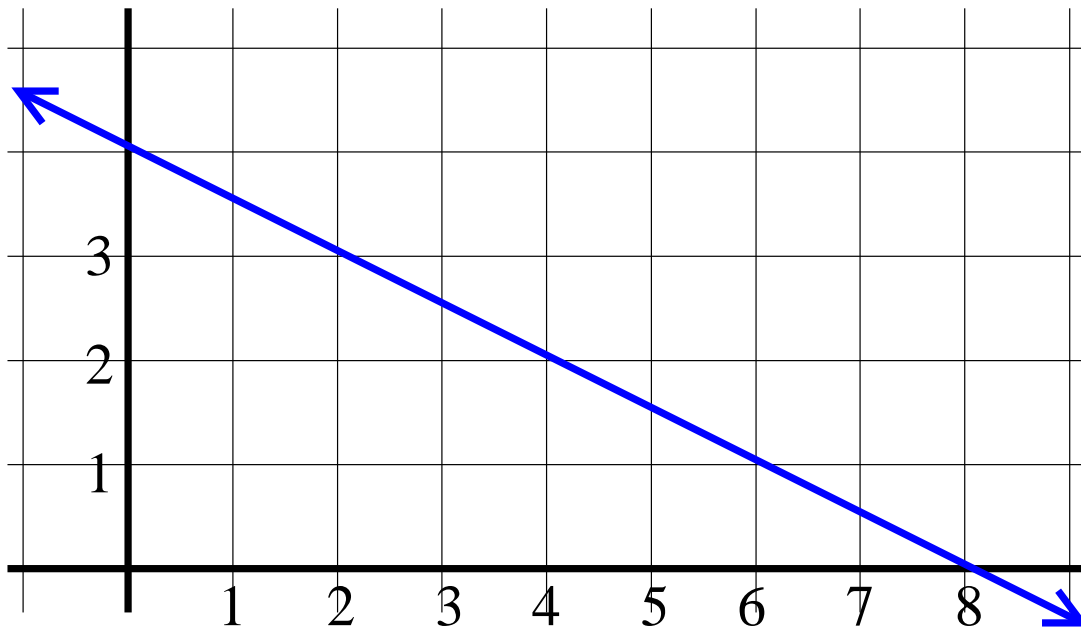


$$f(3)(1) + f(4)(1) + f(5)(1) + f(6)(1) =$$

$$= \left[-\frac{1}{2}(3) + 4\right](1) + \left[-\frac{1}{2}(4) + 4\right](1) \\ + \left[-\frac{1}{2}(5) + 4\right](1) + \left[-\frac{1}{2}(6) + 4\right](1)$$

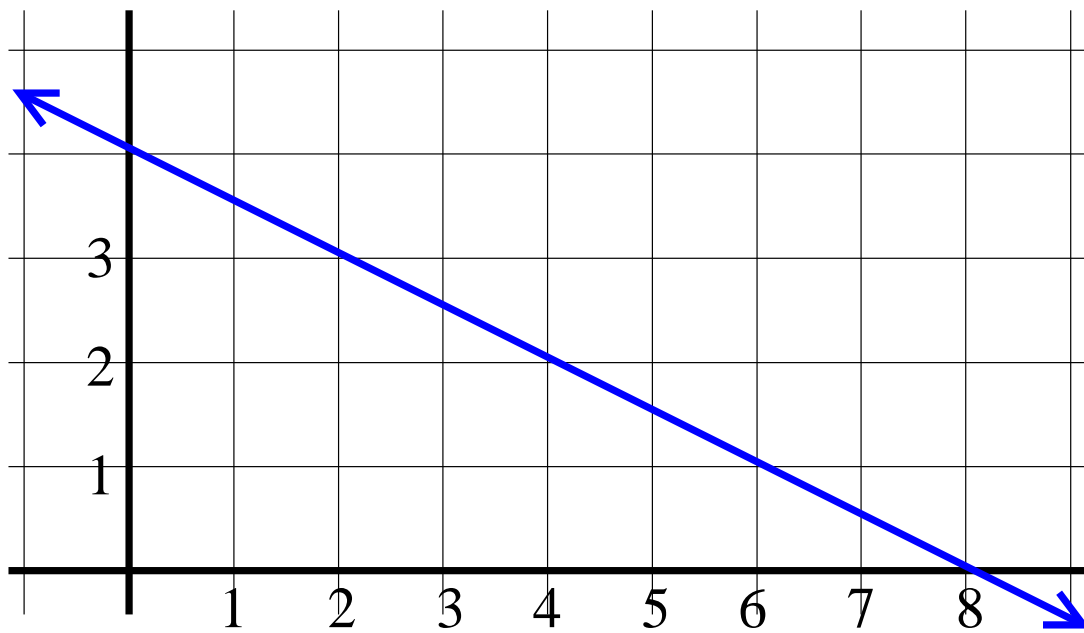
$$= \frac{5}{2}(1) + 2(1) + \frac{3}{2}(1) + 1(1) = 7$$

Inscribed rectangles with $\Delta x = \frac{1}{2}$:

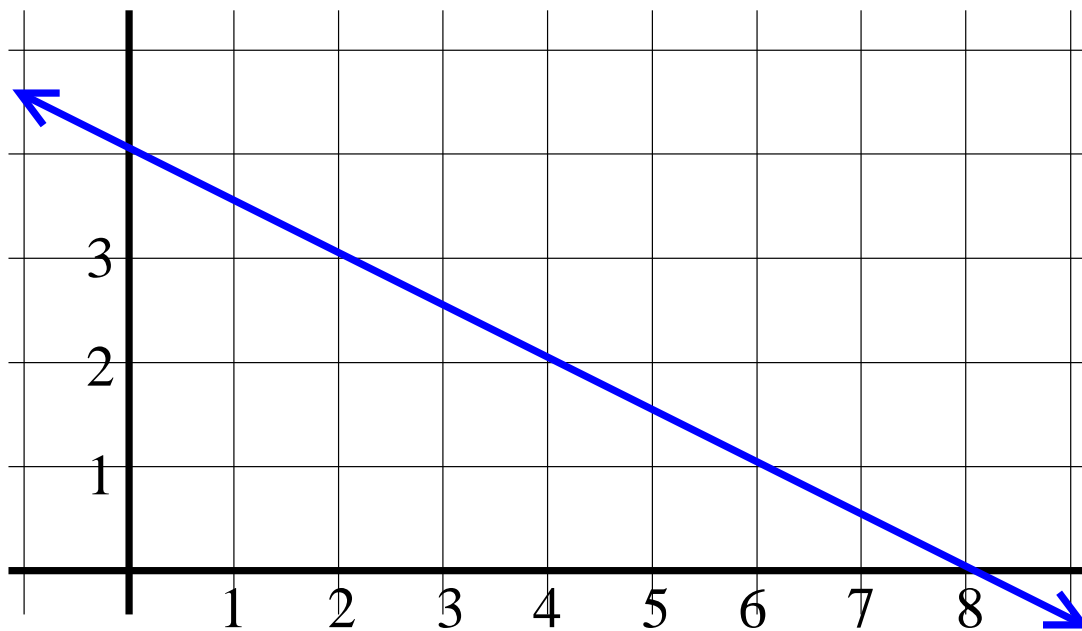


$$\begin{aligned} & f\left(\frac{5}{2}\right)\left(\frac{1}{2}\right) + f(3)\left(\frac{1}{2}\right) + f\left(\frac{7}{2}\right)\left(\frac{1}{2}\right) + f(4)\left(\frac{1}{2}\right) \\ & \quad + f\left(\frac{9}{2}\right)\left(\frac{1}{2}\right) + f(5)\left(\frac{1}{2}\right) + f\left(\frac{11}{2}\right)\left(\frac{1}{2}\right) + f(6)\left(\frac{1}{2}\right) \\ &= \left[-\frac{1}{2}\left(\frac{5}{2}\right) + 4\right]\left(\frac{1}{2}\right) + \left[-\frac{1}{2}(3) + 4\right]\left(\frac{1}{2}\right) + \left[-\frac{1}{2}\left(\frac{7}{2}\right) + 4\right]\left(\frac{1}{2}\right) \\ &+ \left[-\frac{1}{2}(4) + 4\right]\left(\frac{1}{2}\right) + \left[-\frac{1}{2}\left(\frac{9}{2}\right) + 4\right]\left(\frac{1}{2}\right) + \left[-\frac{1}{2}(5) + 4\right]\left(\frac{1}{2}\right) \\ &+ \left[-\frac{1}{2}\left(\frac{11}{2}\right) + 4\right]\left(\frac{1}{2}\right) + \left[-\frac{1}{2}(6) + 4\right]\left(\frac{1}{2}\right) \\ &= \frac{11}{4}\left(\frac{1}{2}\right) + \frac{5}{2}\left(\frac{1}{2}\right) + \frac{9}{4}\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right) + \frac{7}{4}\left(\frac{1}{2}\right) + \frac{3}{2}\left(\frac{1}{2}\right) + \frac{5}{4}\left(\frac{1}{2}\right) + 1\left(\frac{1}{2}\right) \\ &= \frac{15}{2} \end{aligned}$$

Inscribed rectangles with $\Delta x = \frac{6-2}{n} = \frac{4}{n}$:



Circumscribed rectangles with $\Delta x = 1$:



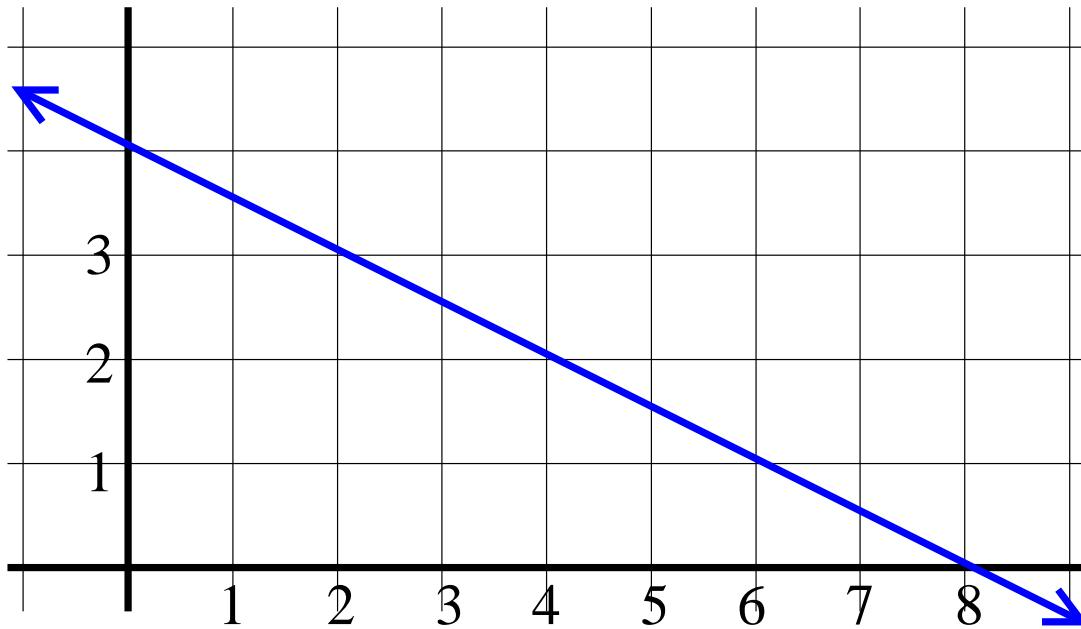
$$f(2)(1) + f(3)(1) + f(4)(1) + f(5)(1) =$$

$$= \left[-\frac{1}{2}(2) + 4\right](1) + \left[-\frac{1}{2}(3) + 4\right](1) \\ + \left[-\frac{1}{2}(4) + 4\right](1) + \left[-\frac{1}{2}(5) + 4\right](1)$$

$$= 3 + \frac{5}{2}(1) + 2(1) + \frac{3}{2}(1) = 9$$

Estimate the distance traveled between $t = 2$ and $t = 6$ if the velocity is given by the function $f(t) = -\frac{1}{2}t + 4$.

Estimate using inscribed rectangles with $\Delta t = 1$:



$$\begin{aligned} & f(3)(1) + f(4)(1) + f(5)(1) + f(6)(1) = \\ & = \left[-\frac{1}{2}(3) + 4\right](1) + \left[-\frac{1}{2}(4) + 4\right](1) \\ & \quad + \left[-\frac{1}{2}(5) + 4\right](1) + \left[-\frac{1}{2}(6) + 4\right](1) \\ & = \frac{5}{2}(1) + 2(1) + \frac{3}{2}(1) + 1(1) = 7 \end{aligned}$$

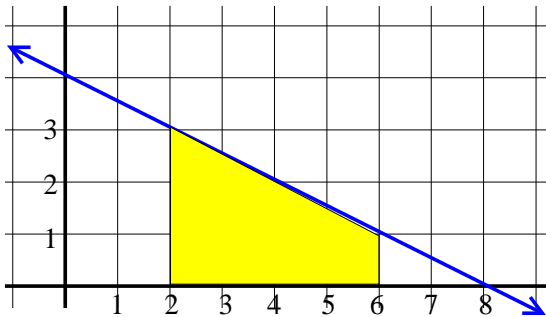
Defn: $\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$

If f is continuous, can use inscribed rectangles, circumscribed rectangles, all left-hand endpoints, all right-hand endpoints, or all midpoints, etc.

If $\Delta x = \frac{b-a}{n}$ and if right-hand endpoints are used, then $x_i = a + i\Delta x = a + \frac{(b-a)i}{n}$

$$\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(a + \frac{(b-a)i}{n}\right) \left(\frac{b-a}{n}\right)$$

Find the distance traveled between $t = 2$ and $t = 6$ if the velocity is given by the function $f(t) = -\frac{1}{2}t + 4$.



Method 1: In this case our function is very simple, so we can determine the area without calculus:

Method 2: Use calculus by estimating with rectangles and taking limit.

$$\begin{aligned}
 \text{Area} &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(a + \frac{(b-a)i}{n}\right) \left(\frac{b-a}{n}\right) \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(2 + \frac{4i}{n}\right) \left(\frac{4}{n}\right) \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[-\frac{1}{2}\left(2 + \frac{4i}{n}\right) + 4\right] \left(\frac{4}{n}\right) = 8
 \end{aligned}$$

Method 3 (section 5.3): Use calculus by integrating.

$$\begin{aligned}
 \int_2^6 \left(-\frac{1}{2}t + 4\right) dt &= \left(-\frac{1}{4}t^2 + 4t\right) \Big|_2^6 \\
 &= \left(-\frac{1}{4}(6)^2 + 4(6)\right) - \left(-\frac{1}{4}(2)^2 + 4(2)\right) \\
 &= -9 + 24 - (-1 + 8) = 15 - 7 = 8
 \end{aligned}$$