

Let $s : \{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\} \rightarrow \mathbf{R}$, $s(x) = \sin(x)$.

Note I have chosen the domain of s to be $\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\}$.

Hence the image of s is $\{0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1\}$.

Observe $\{0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1\} = \{0, \frac{\sqrt{1}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{4}}{2}\}$.

Note $0 < \frac{\pi}{6} < \frac{\pi}{4} < \frac{\pi}{3} < \frac{\pi}{2}$.

Note $0 < \frac{1}{2} < \frac{\sqrt{2}}{2} < \frac{\sqrt{3}}{2} < 1$.

Since \sin is an increasing function on the interval $[0, \frac{\pi}{2}]$, we know that

$$\sin(0) = 0, \sin(\frac{\pi}{6}) = \frac{1}{2}, \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}, \sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}, \sin(\frac{\pi}{2}) = 1.$$

Note $\{0 + \frac{\pi}{2}, \frac{\pi}{6} + \frac{\pi}{2}, \frac{\pi}{4} + \frac{\pi}{2}, \frac{\pi}{3} + \frac{\pi}{2}, \frac{\pi}{2} + \frac{\pi}{2}\} = \{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi\}$

Let $S : \{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi\} \rightarrow \mathbf{R}$, $S(x) = \sin(x)$.

Note the image of S is $\{0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1\}$.

Since \sin is a decreasing function on the interval $[0, \frac{\pi}{2}]$, we know that

$$\sin(\frac{\pi}{2}) = 1, \sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}, \sin(\frac{3\pi}{4}) = \frac{\sqrt{2}}{2}, \sin(\frac{5\pi}{6}) = \frac{1}{2}, \sin(\pi) = 0.$$

We can similarly determine other values of the \sin function as well as values of the cosine function.