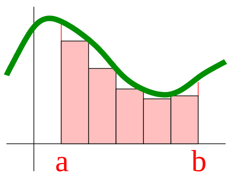
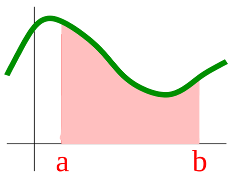
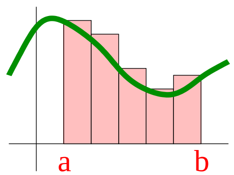
Find the area between $y = 0, \quad y = f(x), \quad x = a, \quad x = b.$
Special case: Suppose f is continuous and $f > 0$.







$$\text{area of } n \text{ inscribed rectangles} \quad \leq \quad \text{actual area} \quad \leq \quad \text{area of } n \text{ circumscribed rectangles}$$

$$\lim_{n\to\infty}\begin{pmatrix}\text{area of } n\\ \text{inscribed}\\ \text{rectangles}\end{pmatrix} \leq \text{actual area} \leq \lim_{n\to\infty}\begin{pmatrix}\text{area of } n\\ \text{circumscribed}\\ \text{rectangles}\end{pmatrix}$$

Theorem:
$$L = \lim_{n\to\infty}\begin{pmatrix}\text{area of } n\\ \text{inscribed}\\ \text{rectangles}\end{pmatrix} = \lim_{n\to\infty}\begin{pmatrix}\text{area of } n\\ \text{circumscribed}\\ \text{rectangles}\end{pmatrix} = U.$$

$$\text{area of } n \text{ inscribed rectangles} \quad \leq \quad \Sigma_{i=1}^n f(x_i)\Delta x \quad \leq \quad \text{area of } n \text{ circumscribed rectangles}$$

where x_i could be right end-point, left end-point, mid-point, or etc.

$$\lim_{n\to\infty}\begin{pmatrix}\text{area of } n\\ \text{inscribed}\\ \text{rectangles}\end{pmatrix} \leq \lim_{n\to\infty}\sum_{i=1}^n f(x_i)\Delta x \leq \lim_{n\to\infty}\begin{pmatrix}\text{area of } n\\ \text{circumscribed}\\ \text{rectangles}\end{pmatrix}$$

Theorem: actual area $= \displaystyle\lim_{n\to\infty}\sum_{i=1}^n f(x_i)\Delta x$

Cor: If f is continuous, $\int_a^b f(x)dx =$ NET area $= \displaystyle\lim_{n\to\infty}\sum_{i=1}^n f(x_i)\Delta x$

Example:

$\int_2^6 (-\frac{1}{2}t + 4)dt = = lim_{n\to\infty}\Sigma_{i=1}^n f(t_i)\Delta t$

$\Delta t = \frac{6-2}{n} = \frac{4}{n}$ (using n equal subintervals)

$t_i = 2 + i\Delta t = 2 + \frac{4i}{n}$ (using right-hand endpoints)

$\int_2^6 (-\frac{1}{2}t + 4)dt = lim_{n\to\infty}\Sigma_{i=1}^n f(2 + \frac{4i}{n})(\frac{4}{n})$

$= lim_{n\to\infty}\Sigma_{i=1}^n[-\frac{1}{2}(2 + \frac{4i}{n}) + 4](\frac{4}{n})$

$= lim_{n\to\infty}\Sigma_{i=1}^n[-1 - \frac{2i}{n} + 4](\frac{4}{n})$

$= lim_{n\to\infty}\Sigma_{i=1}^n[3 - \frac{2i}{n}](\frac{4}{n})$

$= lim_{n\to\infty}\Sigma_{i=1}^n[\frac{12}{n} - \frac{8i}{n^2}]$

$= lim_{n\to\infty}(\Sigma_{i=1}^n \frac{12}{n} - \Sigma_{i=1}^n \frac{8i}{n^2})$

$= lim_{n\to\infty}(12 - \frac{8}{n^2}\Sigma_{i=1}^n i)$

$= lim_{n\to\infty}(12 - \frac{8}{n^2}\frac{n(n+1)}{2})$

$= lim_{n\to\infty}(12 - \frac{4n^2+4n}{n^2})$

$= lim_{n\to\infty}(12 - 4 - \frac{4}{n}) = 8$