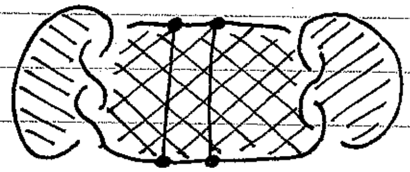
Wednesday, February 11, 2010

Claim: $g(K_1 \# K_2) = g(K_1) + g(K_2)$ (the genus is additive)

Proof: ① $g(K_1 \# K_2) \leq g(K_1) + g(K_2)$

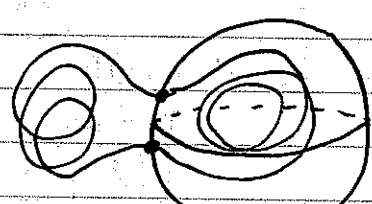


$$g_{K_1 \# K_2} = g_{K_1} + g_{K_2}$$

② $g(K_1 \# K_2) \geq g(K_1) + g(K_2)$

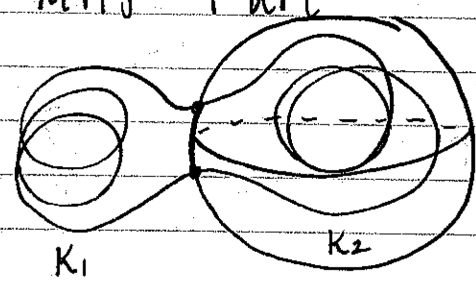
Let S = sphere $\ni S \cap K_1 \# K_2 = 2$ points

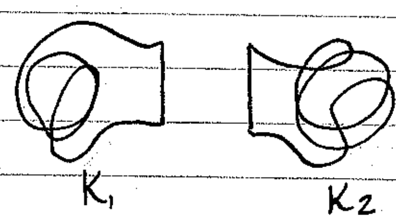
Let M = Seifert surface of $K_1 \# K_2$



Let M have minimum genus among S.S. for $K_1 \# K_2$

(a) Suppose $M \cap S = 1$ arc



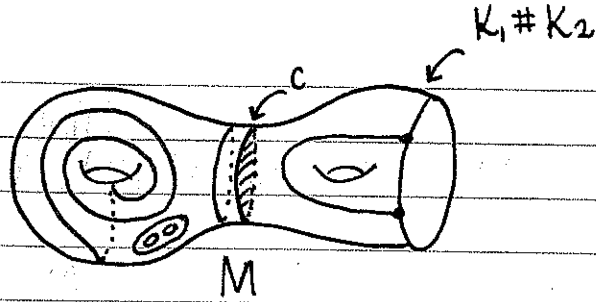


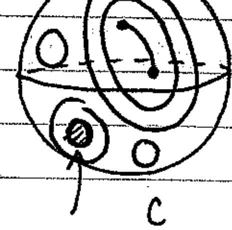
$$g(K_1) + g(K_2) \leq g(M_1) + g(M_2) = g(K_1 \# K_2)$$

(b) $M \cap S = 1$ arc + a finite # of s.c.c.'s

Among S.S. of $K_1 \# K_2$, ~~take~~ which have minimal genus, take one which is minimal wrt # of s.c.c.'s in $M \cap S$. Isotope $M \ni M \pitchfork S$.

$M \cap S = 1$ arc + finite # of s.c.c.'s

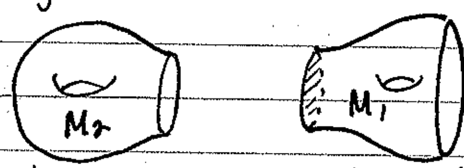




take a s.c.c. C which corresponds to innermost disk on S^2

Possible case: C separates M

Cut along C:



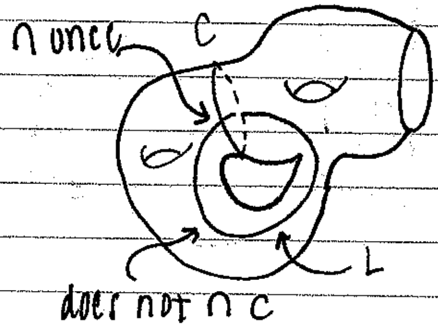
Add disk to component which has $\partial = K_1 \# K_2$

$\partial(M_1 \cup \oslash) = K_1 \# K_2$

$g(M_1 \cup \oslash) \leq g(M)$

$M_1 \cap S$ has fewer s.c.c.'s $\cap$ than $M \cap S$.

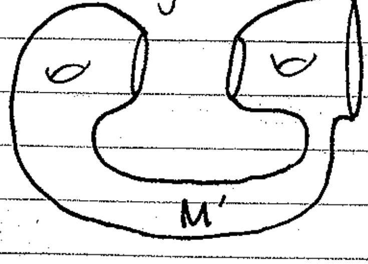
Thus, C does not separate M.



$\Rightarrow \exists$ s.c.c. L as in figure

$|L \cap C| = 1$

Cut along C:

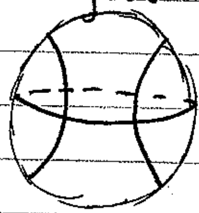


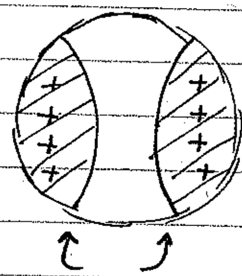
Glue in 2 copies of disks (from S^2)

$g(M') < g(M)$ $\rightarrow\!\times\!\leftarrow$ ∎

Cyclic coverings
References in Rolfsen: 5C, 6AB, (10CD) ← branched

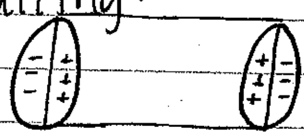
<u>Ex</u>: Tangles





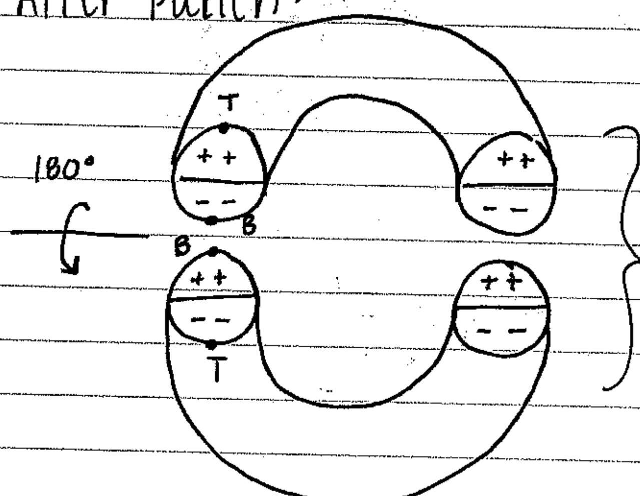
cut along these lines

After cutting:

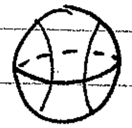


punch up

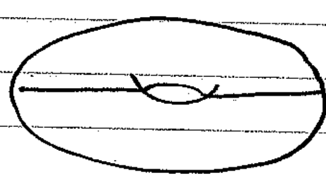
After punch:



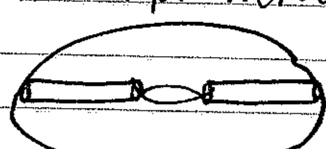
Double cover

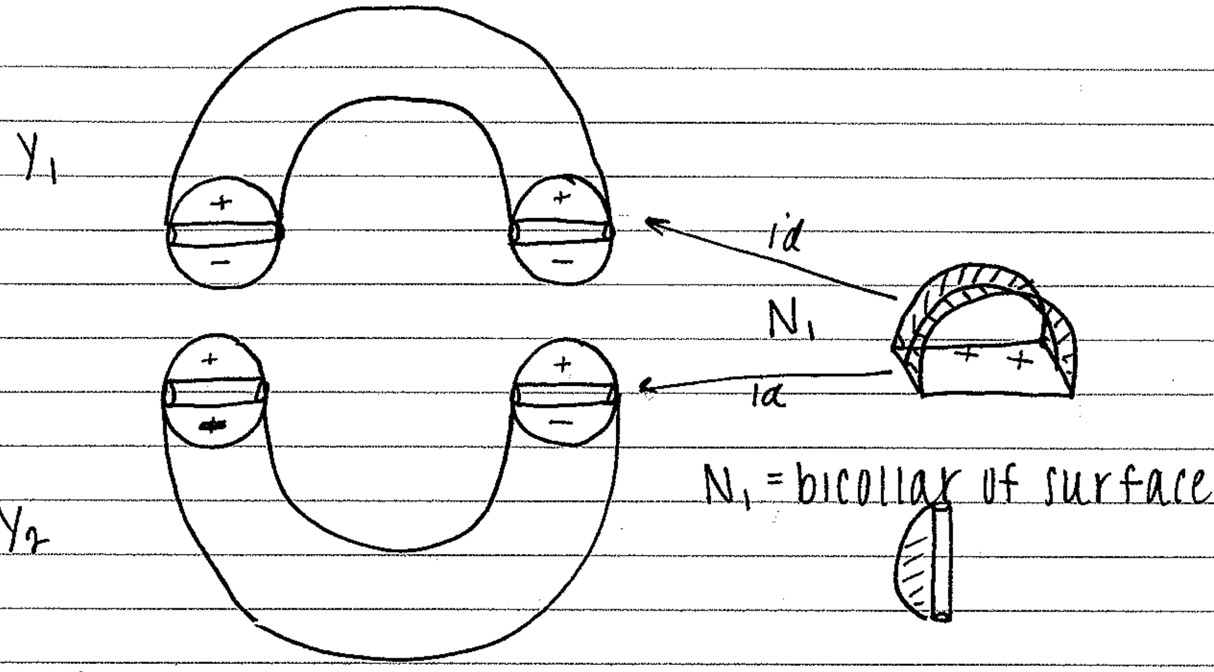
Glue to form double cover of  branched over) (

single copy of strings
2 copies of everything else.



Double cover of complement of strands of tangle:





Triple cover: