2.4 Solve Bernoulli's equation,

$$y' + p(t)y = g(t)y^n,$$

when $n \neq 0, 1$ by changing it

$$y^{-n}y' + p(t)y^{1-n} = g(t)$$

when $n \neq 0, 1$ by changing it to a linear equation by substituting $v = y^{1-n}$

Solve $ty' + 2t^{-2}y = 2t^{-2}y^5$

$ty^{-5}y' + 2t^{-2}y^{-4} = 2t^{-2}$

Let $v = y^{-4}$. Thus $v' = -4y^{-5}y'$

$-4ty^{-5}y' - 8t^{-2}y^{-4} = -8t^{-2}$

$tv' - 8t^{-2}v = -8t^{-2}$

Make coefficient of $v' = 1$

$v' - 8t^{-3}v = -8t^{-3}$

An antiderivative of $-8t^{-3}$ is $4t^{-2}$

Multiply equation by $e^{4t^{-2}}$

$e^{4t^{-2}}v' - 8t^{-3}e^{4t^{-2}}v = -8t^{-3}e^{4t^{-2}}$

$(e^{4t^{-2}}v)' = -8t^{-3}e^{4t^{-2}}$ by PRODUCT rule.

$\int (e^{4t^{-2}}v)'dt = -8\int t^{-3}e^{4t^{-2}}dt$

$e^{4t^{-2}}v = -8\int t^{-3}e^{4t^{-2}}dt.$

Let $u = 4t^{-2}$. Then $du = -8t^{-3}dt$

$e^{4t^{-2}}v = \int e^u du = e^u + C$

$e^{4t^{-2}}v = e^{4t^{-2}} + C$

$v = 1 + Ce^{-4t^{-2}}$

$y^{-4} = 1 + Ce^{-4t^{-2}}$ implies $y = \pm(1 + Ce^{-4t^{-2}})^{-\frac{1}{4}}$

$y' + \frac{2}{t-3}y = 1$

An anti-derivative of $\frac{2}{t-3} = 2ln(t-3)$

$e^{2ln(t-3)} = e^{ln[(t-3)^2]} = (t-3)^2$

$y' + \frac{2}{t-3}y = 1$

$(t-3)^2y' + 2(t-3)y = (t-3)^2$

$\int [(t-3)^2y]' = \int (t-3)^2$

$(t-3)^2y = \frac{(t-3)^3}{3} + C$ implies $y = \frac{(t-3)}{3} + C(t-3)^{-2}$