

Solving first order differential equation:

Method 1 (sect. 2.2): Separate variables.

Method 2 (sect. 2.1): If linear $[y'(t) + p(t)y(t) = g(t)]$, multiply equation by an integrating factor

$$u(t) = e^{\int p(t)dt}.$$

$$\begin{aligned}y' + py &= g \\y'u + upy &= ug \\(uy)' &= ug \\\int (uy)' &= \int ug \\uy &= \int ug \\&\text{etc...}\end{aligned}$$

Method 3 (sect. 2.4): Solve Bernoulli's equation,

$$y' + p(t)y = g(t)y^n,$$

when $n > 1$ by changing it to a linear equation by substituting $v = y^{1-n}$

If $v = \frac{dx}{dt}$, can use the following to simplify (especially if there are 3 variables).

$$\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx}$$