

# Math Department Colloquium

February 12, 2008

**Speaker:** Caroline Klivans

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**Title:** A simplicial matrix tree theorem

**Abstract:** The classical matrix tree theorem expresses the number of spanning trees of a graph in terms of the graph Laplacian. This is a fundamental theorem in combinatorics and has applications in a variety of fields.

We extend the scope of the matrix-tree theorem from graphs to simplicial complexes. The combinatorial Laplacian takes the role of the graph Laplacian and we obtain a simplicial matrix tree theorem which enumerates trees with a homological weighting. The enumeration can be stated in terms of the determinant or the eigenvalues of the Laplacian operator.

By introducing indeterminate forms we obtain finer enumerative information for specific classes of complexes. In particular we investigate the class of shifted complexes which have all integer Laplacian eigenvalues. In this case we give a combinatorial interpretation for the eigenvalues and show particularly nice factorizations of the spanning tree enumerator.