22M:025 Midterm II Sample Problems

In the following, you may use your graphing calculator to verify your conclusions, but you must show the underlying calculus done "by hand".

1. For the function $f(x) = 4x^3 - 12x^2 + 12x + 1$ determine the intercepts, the intervals in which the function is increasing, the intervals in which the function is decreasing, the relative extrema, the intervals in which the function is concave up, the intervals in which the function is concave down, the points of inflection, and any horizontal or vertical asymptotes. Use the information to graph the function.

[Added Note: the intercepts don't work out well on this one...]

2. Let $f(x) = 2x^2 + 1, a = 1, b = 3,\ N = 100,\ \Delta x = \frac{b-a}{N}$.

- **a**. Use your TI to approximate $\int_1^3 (2x^2 + 1)dx$ using both the right and left Riemann sums associated with the regular partition $P = \{a = x_0, x_1, \ldots, x_{100} = b\}$ and $x_i = a + i * \Delta x$.
- **b**. Use the indefinite integral and the Fundamental Theorem of Calculus to find the exact value of $\int_1^3 (2x^2 + 1)dx$.

3. Evaluate the following:

- **a**. $\int (x\sqrt{x} + e^x)dx$
- **b**. $\int x\sqrt{x+1}\,dx$

4. Evaluate the following:

- **a**. The area of the region between the graph of $f(x) = x^2 + 1$ and the x −axis on the interval $[-3, 3]$.
- **b**. $\int_e^{e^2} \frac{1}{x}dx$

5. Let $f(x) = 2x + 1, a = 1, b = 3,\ N = 100,\ \Delta x = \frac{b-a}{N}$. Find the associated right Riemann $R(P)$ sum using the regular partition $P = \{x_0, x_1, \ldots, x_{100}\}$ with $x_i = a + i * \Delta x$. Also find the associated left Riemann sum $R(P)$. What if anything can be said about the relationship between $\int_a^b f(x)dx$, $L(P)$ and $R(P)$?

6. Evaluate the following derivatives.

- **a**. $\frac{d}{dx}\int_0^x t^2 \sin t dt$
- **b**. $\frac{d}{dx}\int_0^{x^3} t \sin t dt$

7. Evaluate the following:

- **a**. $\int \cos x dx$
- **b**. $\int [2 + 3x + x^5]dx$
- **c**. $\int \sec^2 x dx$
- **d**. $\int \frac{1}{x}dx$

8. Use indefinite integrals to evaluate the following:

- **a**. $\int_a^b x^r dx\ (r \neq -1)$
- **b**. $\int_a^b \frac{1}{x}dx$
- **c**. $\int_{a^2}^{b^2} \sqrt{x}\,dx$

9. Evaluate the following integrals.

a. $\int_1^3 [\frac{1}{x} + e^x + xe^{x^2}]dx$

b. $\int_1^3 \frac{2x+1}{x^2+x+2}dx$

10. Evaluate the following integrals.

a. $\int_1^3 \sqrt{1+x}\,dx$

b. $\int \cos 2x dx$

c. $\int x\sqrt{x+1}\,dx$

11. Note that if $c(b-a) = \int_a^b f(x)dx,$ then it is reasonable to think of c as the average value of $f(x)$ on the interval $[a,b]$. For example, if $s(t)$ denotes the distance traveled at time $t,$ and $v(t)$ the velocity at time $t,$ then $s'(t) = v(t),$ so $\frac{\int_a^b v(t)dt}{b-a} = \frac{s(b)-s(a)}{b-a}$ which is the average velocity, or the average value of $v(t)$ on $[a,b]$. Find the average value of the function $f(x) = x^2 + 3x$ on the interval $[1,5]$.

12. Find the area of the region between the graph of $f(x) = x^2 - 1$ and the x –axis on the interval $[-3,3]$.

13. Find the area of the region between the graphs of $y = \sin x$ and $y = \cos x$ on the interval $[0,\pi]$.

14. Let R be the region bounded by the curves $y = x$ and $y = x^2$. Find the volume of the solid generated by rotating the region about (1) the x-axis, and (2) the y-axis.

15. (5.2 #10) Evaluate the following indefinite integral $\int (2x\sqrt{x} - \frac{1}{\sqrt{x}})\,dx$

16. (5.3 #26) Find the following sum $\sum_{k=1}^{100}(2k-1)^2$

17. (5.4 #39) Compute the Riemann sum $\sum_{i=1}^{n} f(x_i^*)\Delta x$ for the function $f(x) = \cos x$ and a regular partition of the interval $[0,\pi]$ into 6 subintervals $[x_{i-1}, x_i]$.

18. (5.5 #12) Evaluate the definite integral $\int_0^1 x^{99}dx$

19. (5.6 #10) Find the avarage value of the function $y = \sin 2x$ on the interval $[0,\pi/2]$.

20. (5.7 #30) Evaluate the indefinite integral $\int \frac{x}{1+x^2}dx$.

21. (5.7 #46) Evaluate the indefinite integral $\int_0^{\pi/2} \sin x \cos x\,dx$.

22. (5.8 #36) Find the area of the region surrounded by the curves, $y = e^{-x}$, $x = 1$ and $y = x + 1$.

23. (6.2 #14) Find the volume of the solid that is generated by rotating around the x-axis the plain region surrounded by the curves $y = e^{-x}$, $y = 0$, $x = 0$ and $x = 1$.

24. (6.3 #8) Use the method of cylindical shells to find the volume of the solid that is generated by rotating around the x-axis the plain region surrounded by the curve $y = x^2$, and lines $y = 2x$, and $y = 5$.

25. (6.4 #22) Find the length of the smooth arc in $y = x - x^3$ from $x = 0$ to $x = 1$.

26. **Review all of the quiz problems**.