

Name: \_\_\_\_\_

**Bayesian Statistics, 22S:138**  
PRACTICE PROBLEMS FOR MIDTERM 1, 2003  
Midterm 1, Fall 2002

Show any computations that you carry out. Use the back of your exam paper if you run out of space.

1. The density of random variable  $X$  is

$$f(x) = 2x, \quad 0 < x < 1$$

- (a) To which parametric family does this density belong (circle one):
- i. beta
  - ii. binomial
  - iii. gamma
  - iv. normal
  - v. Poisson
- (b) What is/are the parameter(s) of the density? (numeric answer)

2. In a particular county in Iowa,

- 54% of registered voters are women
- 46% of registered voters are men

Of men registered voters,

- 55% support the Republican candidate for governor
- 42% support the Democratic candidate for governor
- 3% support the Green party candidate for governor

Of women registered voters,

- 39% support the Republican candidate for governor
- 55% support the Democratic candidate for governor
- 6% support the Green party candidate for governor

- (a) You drew a voter at random from the voter registration list in this county. What is the probability that the voter you drew supports the Green party candidate?

- (b) You are told that the voter you drew supports the Green party candidate. Given that information, what is the probability that the voter you drew is a woman?

3. A statistician believes that the distribution of heights of undergraduate women at UI is normal with known population mean  $\mu = 65$  inches. He wishes to estimate the population *precision* (inverse of the variance) of this distribution. He will use the symbol  $\tau^2$  for the precision.

The sampling density for a single observation  $y$  drawn from this normal distribution is

$$p(y) = \frac{\tau}{\sqrt{2\pi}} \exp\left(-\frac{\tau^2 (y - 65)^2}{2}\right)$$

For data, the statistician will measure two randomly selected UI undergrad women,

- (a) Write the likelihood of  $\tau^2$  if there are two data values, represented symbolically as  $y_1$  and  $y_2$ .

- (b) The statistician puts the following prior on  $\tau^2$ :

$$p(\tau^2) = \text{Gamma}\left(\frac{1}{2}, \frac{9}{2}\right)$$

This prior has as much information as how many prior observations? (In other words, what is the equivalent prior sample size?)

- (c) When the statistician measures his two subjects, their heights are:

$$\begin{aligned} y_1 &= 64 \text{ in} \\ y_2 &= 67 \text{ in} \end{aligned}$$

Write a mathematical expression to which the posterior distribution,  $p(\tau^2|y_1, y_2)$ , is proportional. First write it with the data values represented symbolically as  $y_1$  and  $y_2$ . Then plug in the numeric values.

- (d) To which parametric family does this posterior distribution belong?
- (e) What are the parameters of the posterior distribution (numeric values)?
- (f) What are the mean and variance of this posterior distribution?
- (g) Write the R (or Splus) function that you would use to compute a 95% posterior credible set for  $\tau^2$ . Include the appropriate arguments.
- (h) Suppose that the 95% posterior credible set for  $\tau^2$  turned out to be (0.1, 0.5). What is the correct Bayesian interpretation of this interval?