Midterm Exam I

22M:017

Calculus and Matrix Algebra
for Business

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Let $f(x) = \frac{2}{x}$ and $g(x) = x^2 + 2x - 2$. Find the composite function $f(g(x))$.

a. $\frac{2}{x^2 + 2x - 2}$ b. $\frac{2}{x^2} + \frac{1}{x} - 1$ c. $\frac{4}{x^2} + \frac{4}{x} - 2$

d. $\frac{4}{x^2} + 2x - 2$ e. $\frac{4}{x^2} + \frac{1}{x} - 2$

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Find functions $g(u)$ and $h(x)$ such that $f(x) = g(h(x))$:

$$f(x) = (x-1)^2 - 2(x-1) + 2.$$

a. $g(u) = 2u^2 - u + 2, \quad h(x) = x - 1$
b. $g(u) = 2u^2 - u + 2, \quad h(x) = x + 1$
c. $g(u) = u^2 - u + 2, \quad h(x) = x - 1$
d. $g(u) = u^2 - 2u + 2, \quad h(x) = x - 1$
e. $g(u) = u^2 - 2u + 2, \quad h(x) = x + 1$

Write an equation for the line with the given properties:

Through $(2,-3)$ with slope $-\frac{1}{2}$.

a. $y + 3 = -\frac{1}{2}(x + 2)$
b. $y + 3 = -\frac{1}{2}(x - 2)$
c. $y - 2 = -\frac{1}{2}(x - 3)$
d. $y - 3 = -\frac{1}{2}(x + 2)$
e. $y - 3 = -\frac{1}{2}(x - 2)$

Write an equation for the line through $(1,-4)$ and $(-2,2)$.

a. $y - 4 = -2(x + 1)$
b. $y - 4 = -2(x - 1)$
c. $y + 4 = -2(x - 1)$
d. $y + 4 = 2(x - 1)$
e. $y - 4 = 2(x - 1)$

A closed box with a square base is to have a volume of 200 cubic inches. The material for the top and bottom of the box costs 5 cents per square inch, and the material for the sides costs 2 cents per square inch. Denote by x the length of the base of the box in inches. Express the material cost of the box as a function of x.

a. $10x^2 + \frac{1600}{x}$
b. $5x^2 + \frac{1600}{x}$
c. $5x^2 + \frac{200}{x}$
d. $5x^2 + \frac{8}{x}$
e. $10x^2 + \frac{8}{x}$

A bookstore can obtain a certain gift book from the publisher at a cost of \$5 per book. The bookstore has been offering the book at the price of \$12 per copy, and at this price, has been selling 200 copies a month. The bookstore is planning to lower its price to stimulate sales and estimates that for each \$1 reduction in the price, 20 more books will be sold each month. Express the bookstore's monthly profit from the sale of this book as a function of the selling price x.

a. $(x-5)(200 + 20(x-5))$
b. $(x-12)(200 - 20(x-12))$
c. $(x-5)(200 + 20(12-x))$
d. $(x-5)(200 - 20(12-x))$
e. $(x-12)(200 - 20(12-x))$

Suppose \$1000 is invested at an annual interest rate of 3%. Compute the balance after 10 years if the interest is

compounded monthly.

a. $1000\left(1+\frac{0.3}{4}\right)^{40}$ b. $1000\left(1+\frac{0.03}{4}\right)^{40}$

c. $1000\left(1+\frac{0.3}{12}\right)^{120}$ d. $1000\left(1+\frac{0.03}{12}\right)^{120}$

e. $1000e^{0.3}$

Use the algebraic properties of exponential functions to simplify the expression

$$\frac{5^3}{5^4}.$$

a. 5 b. $\frac{1}{5}$ c. $5^3 - 5^4$

d. $5^{3/4}$ e. $5^3 \cdot 5^4$

Solve the equation for x:

$$3 = e^{0.06x}.$$

a. $\frac{3}{e^{0.06}}$ b. $\frac{\ln 3}{e^{0.06}}$ c. $\frac{3}{0.06}$

d. $\frac{\ln 3}{\ln 0.06}$ e. $\frac{\ln 3}{0.06}$

Differentiate the function

$$y = \frac{1}{t} - \frac{1}{t^2} + \frac{1}{\sqrt{t}}.$$

a. $\frac{1}{t^2} - \frac{2}{t^3} - \frac{2}{t^{3/2}}$

b. $\frac{1}{t^2} + \frac{2}{t^3} - \frac{2}{t^{3/2}}$

c. $-\frac{1}{t^2} - \frac{2}{t^3} + \frac{1}{2t^{3/2}}$

d. $-\frac{1}{t^2} + \frac{1}{2t^3} + \frac{1}{2t^{3/2}}$

e. $-\frac{1}{t^2} + \frac{2}{t^3} - \frac{1}{2t^{3/2}}$

Differentiate the function

$$y = \frac{x-1}{x+2}.$$

a. 1

b. $\frac{3}{(x+2)^2}$

c. $-\frac{3}{(x+2)^2}$

d. $\frac{1}{(x+2)^2}$

e. $-\frac{1}{(x+2)^2}$

Differentiate the function

$$f(x) = e^{-0.05x}.$$

a. $e^{-0.05x}$

b. $0.05e^{-0.05x}$

c. $-0.05e^{-0.05x}$

d. $\frac{e^{-0.05x}}{0.05}$

e. $-\frac{e^{-0.05x}}{0.05}$

Find the second derivative of the function

$$f(x) = 5x^{10} - 4x^3 - 2x + 1.$$

a. $50x^9 - 12x^2 - 2$
b. $50x^9 - 12x^2 - 2x$
c. $450x^8 - 24x - 2$
d. $450x^8 - 24x$
e. $450x^8$

Let $p(x) = -x^2 + 5x + 100$ be the price at which all x units of a particular commodity will be sold. Find the marginal revenue.

a. $-x^3 + 5x^2 + 100x$
b. $-x^2 + 5x + 100$
c. $-2x + 5$
d. $-6x + 10$
e. $-3x^2 + 10x + 100$

Find an equation for the tangent line to $y = x^2 \ln x$ at $x = 1$.

a. $y = x - 1$
b. $y = 2x - 2$
c. $y = 3x - 3$
d. $y = x$
e. $y = x + 2x\ln x$

Find $\frac{dy}{dx}$ by implicit differentiation:

$$2x^2 + y^2 = xy.$$

a. $\frac{dy}{dx} = \frac{4x - y}{x - 2y}$

b. $\frac{dy}{dx} = \frac{4x + y}{x}$

c. $\frac{dy}{dx} = \frac{4x}{1 - 2y}$

d. $\frac{dy}{dx} = \frac{4x}{2y - 1}$

e. $\frac{dy}{dx} = 4x + 2y$