

22M17, FINAL EXAM, FALL 2002, CAMILLO

1. The domain of $\frac{x}{(x-3)\sqrt{x-1}}$ is:

- a. all x with $x > 3$, and $x \neq 1$.
- b. all x with and $x \neq 0$
- c. all x with $x > 1$, and $x \neq 3$
- d. all x with $x \geq 1$ and $x \neq 3$
- e. all with with $x > 3$

2. If $f(x) = x(x^2 + 1)^2$ then $f'(x) =$

- a. $4x(x^2 + 1)$
- b. $2x(x^2 + 1)$
- c. $4x^2(x^2 + 1)$
- d. $x^2 + 4x + 1$
- e. $(x^2 + 1)(5x^2 + 1)$

3. The derivative of $\ln\left(\frac{x}{1+x}\right) =$

- a. $\frac{x+1}{x}$
- b. $\frac{-1}{(x+1)^2} \ln x + \frac{1}{x(x+1)}$
- c. $\frac{1}{x^2} - \frac{1}{(x+1)^2}$
- d. $\frac{1}{x^2+x}$
- e. $\frac{1}{x^2+1}(1 - \ln x)$

4. If $y = f(x)$ is a function that satisfies $2y^2 + xy = 3x$ then the slope of the line tangent to the graph of $f(x)$ at $x = 1$ and $y = 1$ is

- a. 1
- b. -1
- c. $\frac{3}{5}$
- d. $\frac{2}{5}$
- e. 0

5. $\lim_{x \rightarrow -1} \frac{x^3 + 2x^2 + x}{x+1} =$

a. 0

b. $+\infty$

c. $-\infty$

d. 2

e. $-\infty$

6. If $f(x) = \frac{x^2+1}{x^2-1}$ then the derivative of $f(x)$ equals:

a. $\frac{-4x}{(x^2-1)^2}$

b. $\frac{2x+1}{2x-1}$

c. $4x^3$

d. 1

e. $\ln(x^2 - 1)$

7. The function $f(x) = x^3 + 6x^2 + 12x + 1$ is increasing on:

- a. Always decreasing
- b. Always increasing
- c. Increasing for $x < -2$ and $x > 2$
- d. Increasing for $-2 < x < 2$
- e. Increasing for $1 < x$

8. A fruit grower owns an orchard with 40 apple trees. Each apple tree produces 100 apples. The grower would like to add apple trees to his orchard. For each apple tree that he adds, the yield per tree decreases by two apples. How many additional trees should he plant to maximize the apple production of the orchard?

- a. 3
- b. 4
- c. 5
- d. 6
- e. 8

9. The smallest value attained by the function $x - (x - 1)^2$ on the interval $0 \leq x \leq 2$ is:

a. -2

b. -1

c. $\frac{5}{4}$

d. $-\frac{5}{4}$

c. 2

10. $\lim_{x \rightarrow +\infty} \frac{(2x+1)^2}{3x^2+5} =$

a. $\frac{2}{3}$

b. $\frac{4}{3}$

c. $\frac{4}{9}$

d. $\frac{1}{5}$

e. $\frac{9}{8}$

11. If you invest 1000 and this is compounded monthly at six percent interest per year then the amount you have after two years is:

- a. $1000e^{.12}$
- b. $1000(1.06)^2$
- c. $1000(1.005)^{24}$
- d. $1000(1.06)^{12}$
- e. $1000(1.005)^{12}$

12. The equation of the line tangent to the graph of $y = e^x + x + 2$ at $x = 0$ is:

- a. $y = 2x + e + 2$
- b. $y = (e + 2)x + e + 2$
- c. $y = 2x + 3$
- d. $y = 3x + 2$
- e. $y = 3x + e + 2$

13. $\int x(2x^2 + 1)^4 dx =$

a. $\frac{1}{20}(2x^2 + 1)^5 + C$

b. $\frac{1}{5}(2x^2 + 1)^5 + C$

c. $\frac{1}{5}x^{10} + 4x + C$

d. $4(2x^2 + 1)^3 + C$

e. $16x(2x^2 + 1)^3 + C$

14. $\int x \ln x \, dx =$

a. $x \ln x + x + C$

b. $\frac{1}{2}x^2(\ln x)^2 + C$

c. $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

d. $(\ln x + 1) + C$

e. $\frac{x}{\ln x} + \ln x + C$

15. $\int_1^{e^2} \frac{(\ln x)^2}{x} dx =$

- a. $\frac{1}{e} - 1$
- b. $\frac{7}{3}$
- c. $e^4 - 1$
- d. $\frac{e}{3}$
- e. $\frac{8}{3}$

16. If $\frac{dy}{dx} = \frac{x}{y}$ then, $y =$

- a. $\ln x + C$
- b. e^{x+C}
- c. $x^2 + C$
- d. $\frac{1}{\sqrt{x^2+C}}$
- e. $\sqrt{x^2+C}$

17. The set of all solutions of the system,

i) $x + y + z = 3$, ii) $x + z = 2$, iii) $2x + y + 2z = 5$ is described by:

a. $x = 1, y = 1, z = 1$

b. $x = 0, y = 1, z = 2$

c. $x = 2 - a, y = 1, z = a$ for any number a .

d. $x = 2 - a, y = 1, z = 1 + a$ for any number a .

e. There are no solutions.

18. The inverse of the matrix $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ is:

a. $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

b. $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

c. $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

d. $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

e. $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$