# Introduction to Mathematics Research
## A Survey of Applied Mathematics[1]

### *Summary of Modules*

***Collisions Module***

Accurately simulating the behavior of systems of physical bodies involves determining how the bodies react when they make and break contact. This amounts to calculating impact forces using Newton's Laws of motion. A difficulty in modeling collisions is that they inherently lead to instantaneous changes in velocities and accelerations, making it difficult to apply continuous methods.

In this module we introduce a simple complementarity model for collision resolution. We begin with rigid bodies and observe that either the objects are not in contact (and exerting zero force on one another) or are exerting forces on one another (with zero distance between them). Thus, the product of the 'gap' between objects and the contact forces they exert on one another is always zero. We extend this intuitive complementarity condition to rigid bodies in three dimensions. We also address simulation issues, such as collision detection and solution of large linear systems of equations.

***Traffic Flow Module***

Management of traffic flow in urban environments is important in order to allow for efficient and safe transportation. Understanding how cars interact with one another and how different road conditions affect the flow of traffic can help planners develop efficient and robust traffic networks.

In this module, we examine a continuous model for traffic flow derived from the fundamental principle of conservation of cars. We derive a simple differential equation to describe the variation of traffic density over time. This equation can be solved analytically or numerically for traffic density, and then used to determine traffic volume, average flow rate, and total throughput. We extend our basic model by including a "two-second" rule, through which we require drivers to maintain a safe two-second distance between cars.

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***Subsurface Flow Module***

Fresh water is essential for life on Earth. Since the majority of the Earth's fresh water stores are found underground, understanding subsurface flow is vital in determining the viability and the sustainability of water resources. The application of subsurface flow models, however, is not limited to water; contaminant remediation, waste storage, and land usage are just some of the additional issues subsurface modeling can address.

In this module we introduce fundamental hydrology terminology and concepts and use them to derive a three dimensional transient fluid flow equation. Assuming knowledge of material flow parameters, such as hydraulic conductivity and specific storage, we use finite difference methods to solve for hydraulic head (pressure head) at each 'node' on a predetermined grid. We continue to explore the subject by defining advection and hydrodynamic dispersion in a mathematical context in order to derive the advection-dispersion equation for contaminant flow. Once again, we use a finite difference approach to determine contaminant concentration throughout a region of interest.

***Epidemiology Module***

The study of disease propagation has very practical applications in today's world. With biological threats including avian flu and tuberculosis and the potential for bioterrorist attacks, it is important to understand not only how diseases spread but also what combination of quarantine and vaccination can be used to prevent or slow the spread.

In this module, we introduce a standard SIR epidemiological model. This involves partitioning the general population into separate compartments: Susceptible (S), Infected (I), and Recovered (R). We derive a system of differential equations detailing how individuals move from one compartment to another. For simple cases, this system has an analytical solution. As we introduce more compartments (for instance, infected but not yet contagious), the system can only be solved numerically.

***Financial Options Pricing Module***

Financial options are investment vehicles that have value based primarily on an underlying asset. Similar to traditional traded stocks, options can be bought and sold for purely speculative reasons. But unlike stocks, options allow investors to minimize their portfolio risk because the pricing method is entirely deterministic.

In this module, we first develop a stochastic differential equation to model underlying stock (asset) prices. Using this equation and Ito's lemma we derive Black-Scholes partial differential equation for option pricing. After exploring the effect different boundary conditions have on solutions to the equation, we choose to approximate the result (option value) using finite difference methods. We further investigate the topic by introducing applications of Black-Scholes equation to areas such as equity valuation of a company, building of a manufacturing site, or determining the value of a product patent.