CALCULUS FOR BUSINESS, EXAM I, FALL 2003 CAMILLO

1. If $f(x) = \frac{1}{\sqrt{x}} + 1$ and $g(x) = \frac{1}{x}$ then $g(f(x)) =$
a. $\frac{\sqrt{x}}{\sqrt{x+1}}$
b. $\sqrt{x} + 1$
c. $\frac{1}{\sqrt{x}} + 1$
xd. $\frac{\sqrt{x}}{\sqrt{x}+1}$
e. $\frac{\sqrt{x}+1}{\sqrt{x}}$
solution: $g(f(x)) = \frac{1}{\frac{1}{\sqrt{x}}+1}$, multiply the top and bottom by $\sqrt{x}$.
Then, $g(f(x)) = \frac{\sqrt{x}}{1+\sqrt{x}}$

2. Consider $f(x) = \frac{1}{x} + \sqrt{x+2}$. What is the domain of $f(x)$?
a. All x with $x \neq 0$.
b. All x with $x \geqslant 2$.
c. All x with $x > 0$.
d. All x with $x \geqslant -2$ and $x \neq 0$
e. All x with $x \neq 0$ and $x \neq -2$

solution: we need $x + 2 \geqslant 0$, so $x \geqslant -2$, except $x \neq 0$, or else we would be dividing by 0.

3. A rectangular box with a square base and a closed top and bottom has surface area of 60 square feet. Let x be the length of the side of the square base. Find the volume of the box, V, as a function of x.

a. $60x - \frac{x^3}{2}$
b. $30x - x^2$
c. $30x - x^3$
d. $\frac{240}{x} + 2x^2$
xe. $15x - \frac{x^3}{2}$

solution: Let y be the height. Then the area=A = 2(area base)+4(area of a side) $= 2x^2 + 4xy = 60$.

The volume=V = (area base)xheight= $x^2y = V$. We solve for y in the first equation, first $4xy = 60 - 2x^2$

so $y = \frac{1}{4x}(60 - 2x^2)$. Now we substitue for y in the formula for V and get $V = (x^2)\frac{1}{4x}(60 - 2x^2) = (x)\frac{1}{4}(60 - 2x^2) = 15x - \frac{x^3}{2}$

4. If $y = f(x)$ is a function that satisfies $x^2y + y^2x = 2x$, find $\frac{dy}{dx}$ at the point $x = 1$ and $y = 1$.

a. $\frac{2}{3}$
xb. $-\frac{1}{3}$
c. $\frac{1}{3}$
d. $\frac{1}{2}$
e. $-\frac{2}{3}$

solution: Use the product rule: $(x^2)\,'y + x^2y\,' + (y^2)\,'x + y^2(x)\,' = (2x)\,'$

This gives: $2xy + x^2y\,' + 2yy'x + y^2 = 2$ Now we substitutue $x = 1$ and $y = 1$,

so that $2 + y\,' + 2y' + 1 = 2$ Then, collecting terms, $3y\,' = -1$, so $y\,' = \frac{-1}{3}$.

5. $\underset{x\to 5}{Lim}(\frac{2x-10}{25-x^2}) =?$
xa. $-\frac{1}{5}$
b. $\frac{1}{5}$
c. 5
d. -5
e. $\frac{1}{10}$
Factor the numerator and the denominator so the answer $= \underset{x\to 5}{Lim}\frac{2(x-5)}{(5-x)(5+x)}$
$= \underset{x\to 5}{Lim}\frac{-2(5-x)}{(5-x)(5+x)} = \underset{x\to 5}{Lim}\frac{-2}{(5+x)} = \frac{-1}{5}$

6. $\underset{x\to 4}{Lim}(\frac{x-4}{\sqrt{x}-2}) =$
xa. 4
b. -4
c. $\frac{1}{4}$
d. -$\frac{1}{4}$
e. $\frac{1}{2}$
Solution: Rationalize by multiplying the numerator and denominator by $\sqrt{x}+2$.

This gives $\underset{x\to 4}{Lim}(\frac{x-4}{\sqrt{x}-2}) = \underset{x\to 4}{Lim}\frac{(x-4)(\sqrt{x}+2)}{(\sqrt{x}-2)(\sqrt{x}+2)} = \underset{x\to 4}{Lim}\frac{(x-4)(\sqrt{x}+2)}{(x-4)} = \underset{x\to 4}{Lim}\sqrt{x}+2) = 4$

7. $\underset{x\to+\infty}{Lim}\frac{x^2+1}{3x^2+x}$ =?
a. $+\infty$
b. -∞
c. $\frac{2}{3}$
xd. $\frac{1}{3}$
e. 1
Solution $\underset{x\to+\infty}{Lim}\frac{x^2+1}{3x^2+x} = \underset{x\to+\infty}{Lim}\frac{x^2(1+\frac{1}{x^2})}{x^2(3+\frac{1}{x})} = \underset{x\to+\infty}{Lim}\frac{(1+\frac{1}{x^2})}{(3+\frac{1}{x})} = \frac{1}{3}$

8. If $f(x) = \frac{x+1}{x}$ and $g(x) = \frac{1}{x}$ then, $f[g(x)]$ =?
a. $1 + \frac{1}{x}$
b. $\frac{x}{1+x}$
c. $x + 1$
d. $1 - \frac{1}{x+1}$
e. x

Solution: $f[g(x)] = \frac{g(x)+1}{g(x)} = \frac{\frac{1}{x}+1}{\frac{1}{x}}$. Now multiply the top and bottom by x. $\frac{\frac{1}{x}+1}{\frac{1}{x}} = x + 1$.

9. For what value of A is the following function continuous for every x:
$f(x) = Ax^2$ if $0 \leq x < 1$, and $f(x) = x^3 + 1$ if $1 \leq x$?
a. 0
b. 1
xc. 2
d. 3
e. 4

Solution: We need $\frac{Lim}{x \to 1-} Ax^2 = \frac{Lim}{x \to 1+} (x^3 + 1) = 2$
So, $A1^2 = 2$ and $A = 2$.

10. What is the domain of $\frac{1}{\sqrt{x^2+1}}$?
xa. all real numbers.
b. all real numbers except +1 and −1.
c. all x with $-1 \leq x \leq 1$.
d. all x with $-1 < x < 1$.
e. all x with $x \neq 0$

Solution: There are only two problems to deal with, we don't want to divide by 0 and we don't want to take the square root of a negative number. But $x^2 + 1 > 0$ for all x, so the domain is all real numbers.

11. Suppose $y = f(x)$ is a function that satisfies $\frac{1}{x} - \frac{1}{y} = 2$. What is the slope of the line tangent to the graph of the curve defined above at $x = \frac{1}{4}$ and $y = \frac{1}{2}$.

a. $\frac{3}{4}$
b. -4
xc. 4
d. $\frac{1}{2}$
e. 2

Solution: Let us do this by implicit differentiation even though we could solve for y in terms of x.

We have $x^{-1} - y^{-1} = 2$. Taking derivatives, we have $-x^{-2} + y^{-2}y\ ' = 0$

Then $-\frac{1}{x^2} + \frac{1}{y^2}y\ ' = 0$.

So $y\ ' = +\frac{y^2}{x^2}$. Now let $x = \frac{1}{4}$ and $y = \frac{1}{2}$.and $y\ ' = +4$

12. The equation of the line tangent to the graph of $y = \frac{x+1}{x}$ at $x = 1$ is:

a. $y = 2x - 2$
b. $y = 2x + 1$
c. $y = 2x$
d. $y = x + 1$
xe. $y = -x + 3$

Solution: We use the quotient rule to take the derivative: $y\,' = \frac{x(x+1)\ ' - (x+1)x\ '}{x^2} = \frac{x-x-1}{x^2} = \frac{-1}{x^2}$

Now, let x=1 and get $y\ '(1) = -1$. Going back to the original function, we have when $x = 1$, $y = 2$..

So, using point slope form we have,

$y - 2 = -1(x-1) = -x + 1$..so that $y = -x + 3$

13. If $f(x) = 2(x+1)+1$ then $f\ '(1) =$
a. 1
b. 2
c. 3
d. 4
e. 5
Solution: $f\ '(x) = 2$, so $f\ '(1) = 2$. (just a silly question)

14. The equation of the line tangent to the graph of $y = x + \frac{1}{x}$ at $x = 2$ is:
a. $y = \frac{3}{4}x + \frac{1}{8}$
xb. $y = \frac{3}{4}x + 1$
c. $y = \frac{5}{2}x$
d. $y = x + \frac{3}{4}$
e. $y = 1 - \frac{1}{x^2}$
Solution: We write $y = x + x^{-1}$,. then, $y\ ' = 1 - x^{-2}$, .so $y\ '(2) = 1 - \frac{1}{4} = \frac{3}{4}$, then, $y(2) = 2 + \frac{1}{2} = \frac{5}{2}$,
So by point-slope form, $y - \frac{5}{2} = \frac{3}{4}(x-2)$. then $y = \frac{3}{4}x - \frac{6}{4} + \frac{5}{2} = \frac{3}{4}x + 1$

15. The interval or intervals on which $f(x) = x^3 - 3x + 1$ is decreasing are:
xa. $-1 < x < 1$
b. $-\infty < x < -1$ and $1 < x < \infty$
c. $-1 < x < \infty$
d. $-\infty < x < 1$
e. $-\infty < x < \infty$
Solution: First, $f\ '(x) = 3x^2 - 3 = 3(x-1)(x+1)$. So we check the intervals a) $-\infty < x < -1$ and b) $-1 < x < 1$ and $c)1 < x < \infty$ then, $f\ '(-2) = 9$, $f\ '(0) = -1$, $f'(2) = 9$ So the derivative is negative on a. $-1 < x < 1$ only, and so is decreasing on this interval.

16. If $f(x) = \sqrt{1+x^2}$ then, $f\ '(x) =$
xa. $\frac{x}{\sqrt{1+x^2}}$
b. $\frac{1}{2\sqrt{1+x^2}}$
c. $x\sqrt{1+x^2}$
d. $\frac{2x}{\sqrt{1+x^2}}$
e, 2x$\sqrt{1+x^2}$
Solution We use the chian rule. We start with $f(x) = (1+x^2)^{\frac{1}{2}}$ Then, use the power rule so that
$f\ '(x) = \frac{1}{2}(1+x^2)^{-\frac{1}{2}}(1+x^2)\ ' = \frac{1}{2}(1+x^2)^{-\frac{1}{2}}2x = \frac{x}{\sqrt{1+x^2}}$

17. If $f(x) = x\sqrt{1+x}$ then $f\ '(x) =$

a. $\frac{1}{2\sqrt{1+x}}$

b. $\frac{2+x}{2}\sqrt{1+x}$

c. $\sqrt{1+x}$

xd. $\frac{2+3x}{2\sqrt{1+x}}$

e. $\frac{2+3x}{\sqrt{1+x}}$

Solution: Use the product rule. $f(x) = x(1+x)^{\frac{1}{2}}$,, so $f\ '(x) = (1+x)^{\frac{1}{2}} + x\frac{1}{2}(1+x)^{-\frac{1}{2}} =$

$(1+x)^{\frac{1}{2}} + x\frac{1}{2\sqrt{(1+x)}}$.Now multiply the top and bottom of the first term of $f\ '(x)$ by $2\sqrt{(1+x)}$.

Then, $f\ '(x) = \frac{2(1+x)}{2\sqrt{(1+x)}} + \frac{x}{2\sqrt{(1+x)}} = \frac{2+3x}{2\sqrt{(1+x)}}$

18. The function $f(x) = x^2 - x - 6$ is concave up on which of the following:

a. All x with $-2 < x < 3$

b. All x with $-\infty < x < -2$ or $3 < x < +\infty$

xc. All x with $-\infty < x < +\infty$

d. All x with -$\infty < x < \frac{1}{2}$

e. All x with $\frac{1}{2} < x < +\infty$

Solution: We compute the second derivative, $f\ '(x) = 2x - 1$ and $f\ ''(x) = 2$, so the second derivative is always positive and the graph is always concave up.

19. If $f(x) = \frac{x^2+1}{x+2}$ then, $f\ '(x) =$
a. $\frac{-x^2+2x-1}{(x+2)}$
xb. $\frac{x^2+4x-1}{(x+2)^2}$
c. $200x^{199}$
d. $2x$
e. $\frac{x^2+2x+1}{x+2}$

Solution: Use the quotient rule. $f\ '(x) = \frac{(x+2)(x^2+1)\ '-(x^2+1)(x+2)\ ''}{(x+2)^2} = \frac{(x+2)2x-(x^2+1)}{(x+2)^2} = \frac{x^2+4x-1}{(x+2)^2}$

20. If $f(x) = (1 + x^{10})^{20}$ then $f\ '(x) =$
a. $20(1 + x^{10})^{19}$
b. $10x^9(1 + x^{10})^{20}$
c. $20x^9(1 + x^{10})^{19}$
xd. $200x^9(1 + x^{10})^{19}$
e. $20(1 + x^{10})^{19}$
Solution: Use the product rule, $f\ '(x) = 20(1 + x^{10})^{19}(1 + x^{10})' = 20(1 + x^{10})^{19}10x^9 = 200x^9(1 + x^{10})^{19}$.