Calculus for Business, Exam 2, Fall 2003, Camillo

1. The absolute maximum of $f(x) = x + \frac{1}{x}$ on $\frac{1}{2} \leq x \leq 3$ is:
a. 2
b. $\frac{5}{2}$
xc. $\frac{10}{3}$
d. $\frac{17}{4}$
e. $\frac{26}{5}$

Solution:: Compute the derivative and set it equal to zero. . $f'(x) = 1 - \frac{1}{x^2}$

$= 0$. We solve or x and get
$x = 1$ and $x = -1$. We have to check $x = \frac{1}{2}$, 1, 3.. $f(\frac{1}{2}) = \frac{5}{2}$, $f(1) = 2$ and

$f(3) = \frac{10}{3}$

2. You buy calculus books for \$5 and sell them for \$25. At this price you sell 200 books. For each dollar decrease in price you sell 50 more books. What should the price be to maximize your profit?
a. \$22
b. \$20
xc. \$17
d. \$13
e. \$12

Solution: Let x be the price reduction in dollars. Then the the profit per book is $20 - x$ dollars. The number of books sold is $200 + 50x$. So the profit is $(20 - x)(200 + 50x) = P = 4000 + 800x - 50x^2$. Then $P' = 800 - 100x = 0$. so $x = 8$, and the price should be 17 dollars.

3. You wish to make a rectangular pen to hold 100 square feet. The pen is to be laid out in the North-South and East-West directions. The North-South fence costs \$1 per foot and the East-West fence costs \$4 per foot. How many feet of fence should go in the East-West direction to minimize the cost of the pen?(notice, this problem has been repaired, don't tell anyone!)

a. 20
xb. 5
xc. 10
d. 25
e. 4

Solution: Let x be the length of one side of the east -west fence and y be the length of one side of the north -south fence. Then the cost= $C = 2y + 8x$. From the area condition we have $xy = 100$.So $y = \frac{100}{x}$ We put this into the cost function and obtain, $C = 2\frac{100}{x} + 8x. = 200x^{-1} + 8x.$. Then, $C' = -200x^{-2} + 8 = 0$. We obtain, $\frac{200}{x^2} = 8$. Then, $x^2 = 25$. $x = 5$. Technically the answer is 10, but I accepted 5 because it is so easy to misread total distance as dimension.

4. You invest \$500 at 4% compounded quarterly. The number of dollars you have after five years is:

a. $500(1.02)^4$
b. $500(1.025)^{20}$
c. $500(1.01)^5$
d. $500(1.04)^5$
xe. $500(1.01)^{20}$

Solution: You will receive 1% during each each interest period, and there are 20 interest periods, so the answer is,

$500(1.01)^{20}$

5. You can invest money in an account at 6% interest compounded three times per year. How many dollars should you invest now to have $4000 at the end of four years?

a. $4000(1.02)^{12}$
b. $4000(1.06)^{-3}$
c. $4000(1.06)^{3}$
xd. $4000(1.02)^{-12}$
e. $4000(1.06)^{-12}$

Solution: Let B be the amount of money you start with. Then, you collect 2% during each interest period. There will be 12 interest periods. The equation we want to solve is $4000 = B(1.02)^{12}$,, so $B = 4000(1.02)^{-12}$

6. The amount of kryptonite A(t) measured in grams in a sample is given by $A(t) = 100e^{-2t}$, where t is measured in years. What is the half-life of kryptonite, measured in years? (The half life of a substance is the time after which one half of a sample has decayed.)

a. $2ln2$
b. 2
c. $\frac{2}{\ln 2}$
xd. $\frac{\ln 2}{2}$
e. $-\frac{\ln 2}{2}$

Solution: You don't need to know about half life to do this. We know that we are starting with 100 grams and we want to know how long it will take for 50 grams to be left. That is we want to solve for t, where $50 = 100e^{-2t}$.This gives $\frac{1}{2} = e^{-2t}$,. or, $2^{-1} = e^{-2t}$. We take ln's of both sides. So $\ln 2^{-1} = \ln e^{-2t}$.. Using $\ln a^r = r \ln a$ we have, $-ln2 = -2t \ln e$. But $lne = 1$. So, $t = \frac{ln2}{2}$

7. You put \$1000 in an account in which your funds are compounded continuously at 12% per year. At the end of three years your balance in dollars is:

a. $\frac{1000}{e^{.36}}$

b. $1000e^{.64}$

c. $1000(1.12)^3$

xd. $1000e^{.36}$

e. $1000e^{1.36}$

Solution:. The formula is $A = Pe^{rt}$. Here $r = .12$ and $t = 3.$ and $P = 1000$, so $P =$$1000e^{.36}$

8. If $f(x) = \ln \frac{x+1}{x-1}$ then $f\ '(x) =$

xa. $\frac{-2}{(x+1)(x-1)}$

b. $\frac{-2x}{(x+1)(x-1)}$

c. $\frac{-2}{(x+1)^2(x-1)}$

d. $\frac{x-1}{x+1}$

e. $\frac{x+1}{x-1}$

Solution: We use the chain rule.in the form $[ln(f(x)]' = \frac{1}{f(x)} f\ '(x)$. So for $f(x) = \ln \frac{x+1}{x-1}$

$f\ '(x) = \frac{1}{\frac{x+1}{x-1}} (\frac{x+1}{x-1})\ ' = \frac{x-1}{x+1} \frac{(x-1)1-(x+1)1}{(x-1)^2} = (\frac{x-1}{x+1}) \frac{x-1-x-1}{(x-1)^2} = \frac{-2}{(x+1)(x-1)}$

9. If $f(x) = e^{\sqrt{x}}$, then, $f\ '(x) =$

xa. $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$

b. $\frac{e^{\sqrt{x}}}{\sqrt{x}}$

c. $e^{\sqrt{x}}$

d. $\sqrt{x}e^{\sqrt{x}}$

e. $\frac{1}{e^{\sqrt{x}}}$

Solution: We use $(e^{f(x)})' = (e^{f(x)})f\ '(x)$. So, $f\ '(x) = e^{\sqrt{x}}(x^{\frac{1}{2}})' = e^{\sqrt{x}}\frac{1}{2}x^{-\frac{1}{2}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

10. If $f(x) = \frac{\ln x}{e^x}$ then $f\ '(x) =$

a. $\frac{1}{xe^x}$

xb. $\frac{1-x\ln x}{xe^x}$

c. $\frac{1-x\ln x}{e^{2x}}$

d. $e^{-x}\ln x + \frac{1}{x}e^{-x}$

e. $\frac{x\ln x-1}{xe^x}$

Solution: We use the quotient rule: $f\ '(x) = \frac{e^x(\ln x)' - (\ln x)(e^x)'}{(e^x)^2} = \frac{\frac{e^x}{x} - (\ln x)(e^x)'}{(e^{2x})}$, Now multiply the top and bottom by x. We get $f\ '(x) = \frac{e^x - x(\ln x)(e^x)'}{x(e^{2x})}$ =(now, it really gets interesting)= $\frac{e^x(1-x(\ln x)}{xe^xe^x}$ now, cancel e^x to get $\frac{1-x\ln x}{xe^x}$.

11. $\int(\frac{1}{\sqrt{x}}+\frac{1}{x})dx =$

a. $2x^{\frac{1}{2}}+\frac{1}{x^2}+C$

b. $\frac{2}{3}x^{\frac{3}{2}}+C$

c. $\ln|\sqrt{x}|-\frac{1}{x^2}+C$

xd. $2x^{\frac{1}{2}}+\ln|x|+C$

e. $\ln\sqrt{x}+\ln|x|+C$

Solution: Write all expressions as powers, $\int(\frac{1}{\sqrt{x}}+\frac{1}{x})dx = \int(x^{-\frac{1}{2}}+x^{-1})dx = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+\ln x+C.$

$=2x^{\frac{1}{2}}+\ln|x|+C$

12.$\int\frac{x^2}{x^3+1}dx =$

a.$ln\left|x^3+1\right|+C$

xb. $\frac{1}{3}ln\left|x^3+1\right|+C$

c. $ln\left|x^2+1\right|+C$

d. $\frac{1}{(x^3+1)^2}+C$

e.$\frac{1}{3(x^3+1)^2}+C$

Solution. Notice that the numerator looks almost like the derivative of the denominator. Let $u = x^3+1$..

Then $du = 3x^2dx$ So we rewrite .$\int\frac{x^2}{x^3+1}dx$ as $\frac{1}{3}\int\frac{3x^2}{x^3+1}dx = \frac{1}{3}\int\frac{du}{u} = \frac{1}{3}\ln u = ln\left|x^3+1\right|+C$

13. $\int 2\sqrt{1+t}dt =$
a. $(1+t)^{\frac{3}{2}} + C$
b. $\frac{1}{3}(1+t)^{\frac{3}{2}} + C$
xc. $\frac{4}{3}(1+t)^{\frac{3}{2}} + C$
d. $2t + \frac{4}{3}t^{\frac{3}{2}}$
e. $-4(1+t)^{-\frac{1}{2}}$
Solution, let $u = 1+t$ so that $du = dt$. Then, $\int 2\sqrt{1+t}dt = \int 2u^{\frac{1}{2}}du = \frac{4}{3}u^{\frac{3}{2}} = \frac{4}{3}(1+t)^{\frac{3}{2}} + C$

14. Solve for y if $\frac{dy}{dx} = 2y$.
xa. $y = e^{2x+C}$
b. $y = \sqrt{x+C}$
c. $y = \ln(|2x+C|$
d. $y = x^2 + C$
e. $y = \frac{1}{x+C}$
Solution: We cross multiply and get $\frac{dy}{y} = 2dx$. Then we integrate and get $\ln|y| = 2x + C$. Then we raise both sides to the power e, and we get $y = e^{2x+C}$.

15. $\int x \ln(x) dx =$
a. $x \ln x - x + C$
xb. $\frac{x^2}{2} \ln x - \frac{1}{4} x^2 + C$
c. $\frac{x}{2} \ln x - \frac{1}{4} x^2 + C$
d. $\frac{x^2}{2} \ln x - \frac{1}{4} x + C$
e. $x \ln x + C$
Solution:, we do this by parts. Let $u = \ln x$ and $dv = x$. Then $du = \frac{1}{x} dx$ and $v = \frac{x^2}{2}$
We have $\int x \ln(x) dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \frac{1}{x} dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$

16. The area below the graph of $y = x^3 + 1$ and above the interval $0 \leq x \leq 2$ is:
a. 2
xb. 4
c. 6
d. 8
e. 12
Solution: We evaluate $\int_0^2 (x^3 + 1) dx = \frac{x^4}{4} + 1$(from 0 to 2, I can't find the vertical bar on my program!)
$= \frac{2^4}{4} + 1 - 1 = 4$

17. $\int_0^1 xe^{x^2}dx =$
a. $\frac{e+1}{2}$
xb.
c. $\frac{e}{2}$
d. $e - 1$
e. $e^2 + 1$
Solution: let $u = x^2$ and $du = 2x$. So $\int xe^{x^2}dx = \frac{1}{2}e^{x^2}$. Then, evaluating from 0 to 1 we get $\int_0^1 xe^{x^2}dx = \frac{1}{2}e^{1^2} - \frac{1}{2}e^{0^2} = \frac{1}{2}e - \frac{1}{2} = \frac{e-1}{2}$(note, $\mathrm{e}^0 = 1$)

18. The value of a certain machine decreases over a ten year period at a rate that changes with time. When the machine is x years old the rate at which the value is changing is $220(x-10)$ dollars per year. By how many dollars does the machine depreciate during the second year?
a. 220
b. 2090
c. 2200
d. 3740
xe.1870
Solution:, when you see a rate integrate, that is not very intellectual but it works. So, the answer is $\int_1^2 220(x-10)dx = 110(x-10)^2$ from 1 t0 2(where is that vertical bar!!!!)= $110(2-10)^2 - 110(1-10)^2$=1870

19. You start measuring the population of a colony of bacteria at 12:00 P.M. The population size is given by $N(t) = 1000e^{2t+1}$ where t is the time in hours after noon. (note, this differs slightly from the usual form exponential growth). The starting population in numbers of bacteria is:

a. 1000

b. 2000

xc. 1000e

d. $1000e^3$

e. $2000e^3$

Solution:. Trick question!!! $N(0)$ is the starting population,.and here $N(t) = 1000e^{2(0)+1} = 1000e^{0+1} = 1000e$

20. $\int_0^1 (1 + x^{100})dx =$

xa. $\frac{102}{101}$

b. 2

c. 100

d. $\frac{100}{99}$

e. $\frac{2^{101}}{101}$

Solution: $\int_0^1 (1 + x^{100})dx = x + \frac{x^{101}}{101}$ evaluated from o to 1 and this equals $1 + \frac{1^{101}}{101} = \frac{102}{101}$