ENGINEERING MATH IV - HW 5 - DUE: 10/12/2007

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Extra questions for HW - 5.

1) Consider the following third order differential equation

$$\frac{d^3y}{dt} + p_1(t)\frac{d^2y}{dt} + p_2(t)\frac{dy}{dt} + p_3(t)y = f(t) \tag{0.1}$$

Introducing the changes,

$$x_1(t) = y(t), x_2(t) = \frac{dy}{dt}(t) \quad \text{and } x_3(t) = \frac{d^2y}{dt}(t)$$

a) Find the 3-dimensional first order system equivalent to (0.1)

b) Put it in matricial form;

c) Assume, $p_1, p_2, p_3, f : (a, b) \to \mathbb{R}$ are continuous functions. As done in class for second order equations, use (a), (b) and Picard's Theorem to state the existence and uniqueness Theorem for equations (0.1) and sketch its proof.

2) Suppose $y_1, y_2 : (a, b) \to \mathbb{R}$ are linearly independent solutions to

$$\frac{d^2y}{dt} + p(t)\frac{dy}{dt} + q(t)y = 0$$

where $p, q : (a, b) \to \mathbb{R}$ are continuous. Assume, $y_1(t)$ never vanishes

a) Compute the following expression $\frac{d}{dt}(\frac{y_2}{y_1})$ in terms of the Wronskian $W(t) = W(y_1, y_2)$.

b) Show that,

$$y_2(t) = y_1(t) \cdot \{\frac{y_2(t_0)}{y_1(t_0)} + \int_{t_0}^{t} \frac{W(t_0)e^{-\int_{t_0}^{\tau} p(s)ds}}{y_1^2(t)} d\tau\}$$

3) If $y_1(t) = t$ is a solution to

$$t^2\frac{dy^2}{dt} + 2t\frac{dy}{dt} - 2y = 0, \quad t > 0$$

Determine a second solution for this equation (Hint: Use Abel-Liouville Theorem)