HW 4 - PRACTICE QUESTIONS FOR MIDTERM 1

INSTRUCTOR: DIEGO MOREIRA

1. Decide if the following functions: $y_1(t) = t^2e^{-2t}, y_2(t) = t^2e^{-2t} + 150e^{-2t}, y_3 = e^{3t} + 7, y_4 = e^{-2t}(t^2 + \pi), y_5(t) = cos(t) + 3$ are solutions to

$$\frac{dy}{dt} + 2y = 2te^{-2t}$$

Justify your answer.

Solve the following differential equations in questions 2 - 6, by finding its general solution.

2. $ty^{'} + 2y = sin(t)$;
3. $y^{'} = \dfrac{2x}{1+2y}$;
4. $y^{'} + 3y = y^3$;
5. $y^{'} = \dfrac{x+3y}{x-y}$;
6. $e^x sin(y) + 3y + (e^x cos(y) + 3x)\dfrac{dy}{dx} = 0$;
7. Solve the following IVP

$$\begin{cases} \dfrac{dy}{dt} + t^2y = t^2y^4 \\ y(0) = (\frac{1}{2})^{1/3}, \end{cases} \tag{0.1}$$

8. Let $y(t)$ be any solution to

$$\frac{dy}{dt} - 2y = e^{-3t}$$

Show, that $y(t) \to -\infty$ as $t \to -\infty$

9. Solve $y^{'} - 2y = t^2e^{2t}$ by variation of parameters technique.
10. Solve $y^{'} + \dfrac{1}{t}y = 3cos(t)$ also by variation of parameters technique.
11. Solve $ydx + (2x - ye^y)dy = 0$

12. Solve the IVP below, with discontinuous coefficients

$$y^{'} + p(t)y = 1, y(0) = 0$$

where

$$p(t) = \begin{cases} 0, & 0 \leq t \leq 1 \\ t, & t > 1 \end{cases} \tag{0.2}$$

Is the solution continuous at $t = 0$? Is it differentiable at $t = 0$? Explain. (Hint: For the question about differentiability, you might consider to use L'Hospital rule).

13. Consider the following equation

$$\frac{dy}{dt} + p(t)y = g(t) \tag{0.3}$$

Show that:

- If $g \equiv 0$ and φ_1, φ_2 are solutions to (0.3), then $\varphi_3(t) := \varphi_1(t) + \varphi_2(t)$ is also a solution to (0.3).
- If g_1 and g_2 are solutions to (0.3), then $\varphi_4(t) := \varphi_1(t) - \varphi_2(t)$ is a solution to

$$\frac{dy}{dt} + p(t)y = 0$$

14. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $|f(x)| \leq M$ and $|\frac{df}{dx}(x)| \leq N$, $\forall x \in \mathbb{R}$, $M, N > 0$. Assume also, that $f(0) = 0$. Determine all the solutions to

$$\begin{cases} \dfrac{dy}{dt} = f(y) \\ y(0) = 0, \end{cases} \tag{0.4}$$

15. Consider the following initial value problem

$$\begin{cases} \dfrac{dy}{dt} = y^{1/3}, t \geq 0 \\ y(0) = 0, \end{cases} \tag{0.5}$$

Using the fact that this equation is separable, find at least 3 different solutions to the IVP above. Explain why this does not violate the uniqueness in Picard's Theorem.