Lagrange Multiplier Problems

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1 Problems

Problem 1) Let $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = ax + by$, where $a^2 + b^2 \neq 0$.

a) Find the critical points of f along the unit circle $S^1 := \{(x, y) \in \mathbb{R}^2;\ \ x^2 + y^2 = 1\}$

b) Find the extreme values (Max, Min values) of f in the circle S^1.

Problem 2) Let $\mathcal{C}$ be a level curve of a function $f : \mathbb{R}^2 \to \mathbb{R}$, say, $\mathcal{C} = f^{-1}(0)$. Assume that $\mathcal{C}$ does not pass through the origin $O = (0, 0)$. Show that if A is the point in the curve closest or farthest to the origin, then vector A is normal to the curve $\mathcal{C}$.

Problem 3) Let us consider the function given by $f(x, y) = Ax^2 + 2Bxy + Cy^2$. Show that if $u_0 = (x_0, y_0)$ is an extreme value of f along the circle $S^1 := \{(x, y) \in \mathbb{R}^2;\ \ x^2+y^2 = 1\}$ then u_0 is an eingevector of the matrix M

$$\mathbf{M} = \begin{pmatrix} A & B \\ B & C \end{pmatrix}$$

i.e, there exists a $\lambda \in \mathbb{R}$ such that $M \cdot \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \lambda \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$

Problem 4) Among the points in the ellipse

$$\Sigma = \{(x, y) \in \mathbb{R}^2;\ \ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\}$$

Find the closest one to the origin of $\mathbb{R}^2$, $\ O = (0, 0)$.