Homework 5
Summer 2006

In $\boldsymbol{R}^n$ prove that:

1) $\boldsymbol{x} \cdot \boldsymbol{x} = 0$ if and only if $\boldsymbol{x} = \mathbf{0}_n$.

2) $|\|\boldsymbol{x}\| - \|\boldsymbol{y}\|| \leq \|\boldsymbol{x} - \boldsymbol{y}\|$.

3) $|\|\boldsymbol{x}\| - \|\boldsymbol{y}\|| = \|\boldsymbol{x} - \boldsymbol{y}\|$ if and only if $\boldsymbol{x}$ is orthognal to $\boldsymbol{y}$.

Hint: Use Problem 1 and consider a quadratic polynomial just as we did in the proof of the Cauchy-Schwartz inequality.

4 $\|\boldsymbol{x} + \boldsymbol{y}\|^2 = \|\boldsymbol{x}\|^2 + \|\boldsymbol{y}\|^2$ if and only if $\boldsymbol{x}$ is orthognal to $\boldsymbol{y}$ (that is, $\boldsymbol{x}$ and $\boldsymbol{y}$ are perpendicular) .

5) $\boldsymbol{x}$,$\boldsymbol{y}$ are perpendicular if and only if $\|\boldsymbol{x} + \boldsymbol{y}\|^2 = \|\boldsymbol{x}\|^2 + \|\boldsymbol{y}\|^2$.

6) $|\boldsymbol{x} \cdot \boldsymbol{y}| = \|\boldsymbol{x}\| \|\boldsymbol{y}\|$ if and only if there exists a real number γ such that $\boldsymbol{x} = \gamma \boldsymbol{y}$.

From the text, do exercises 1 through 4 from Section 2.1 (page 37) and all exercises from section 2.2 (page 41).