4.) Circle the general solution to the differential equation whose direction field is given below:

A) $y = t + C$

B) $y = t^2 + C$

C) $y = e^t + C$

D) $y = Ce^t + t + 1$

E) $y = Ce^t$

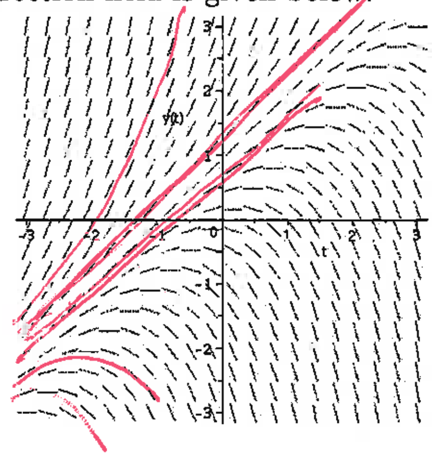
F) $y = e^t + t + C$

G) $y = ln(t) + C$

H) $y = C$

I) $y = sin(t) + C$

J) $y = cos(t) + C$



5.) Which of the following could be the general solution to the differential equation whose direction field is given below:

A) $y = t + C$

B) $y = t^2 + C$

C) $y = e^t + C$

D) $y = \frac{(t-1)^3}{3} + C$

E) $y = Ce^t$

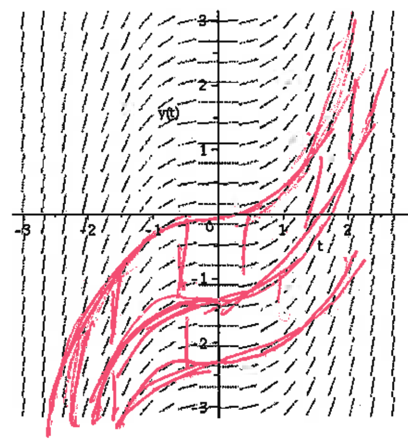
F) $y = \frac{t^3}{3} + C$

G) $y = ln(t) + C$

H) $y = C$

I) $y = \frac{Ct^3}{3}$

J) $y = \frac{C(t-1)^3}{3}$



6.) Circle the differential equation whose direction field is given below:

A) $y' = t^2$

B) $y' = y + 3$

C) $y' = e^t$

D) $y' = t + 1$

E) $y' = t - y$

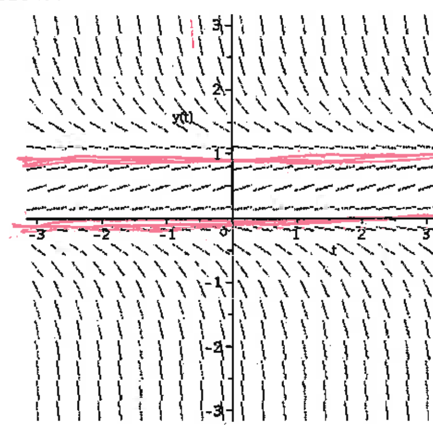
F) $y' = y - t$

G) $y' = (1 + y)(1 - y)$

H) $y' = y(1 + y)$

I) $y' = t(1 - t)$

J) $y' = y(1 - y)$



$y = +1$

$y = 0$

$y = 0$

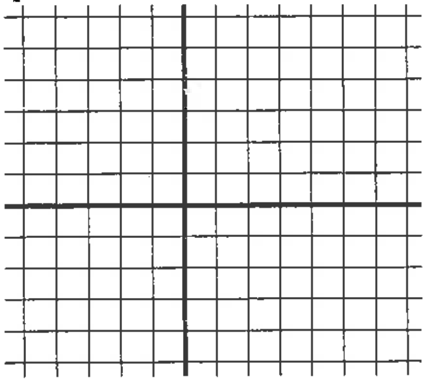
$y = +1$

Ex. $y' = f(y)$

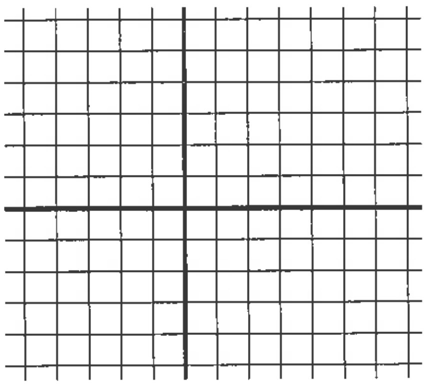
8.1 supplemental HW

1.) For each of the following differential equations (i) draw its direction field; (ii) sketch the solution of the direction field that passes through the point (-2, 1); (iii) state the general solution to the differential equation.

a.) $y' = 0$



b.) $y' = -1$



$y = t + 1$

2.) Circle a solution to the differential equation whose direction field is given below:

~~A) $y = t^2$~~

B) $y = \frac{1}{2}t + 1$

~~C) $y = e^t$~~

D) $y = t + 1$

~~E) $y = -2e^t$~~

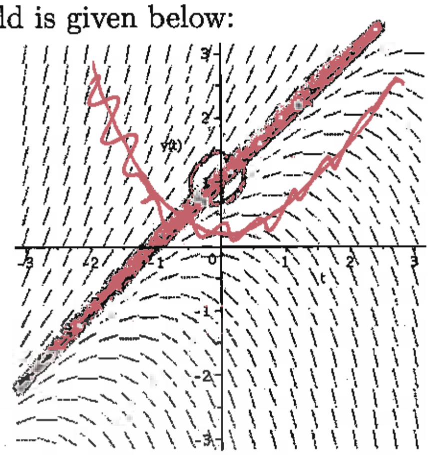
F) $y = 2t + 1$

~~G) $y = ln(t)$~~

~~H) $y = 0$~~

~~I) $y = sin(t)$~~

~~J) $y = cos(t)$~~



3.) Circle the differential equation whose direction field is given below:

~~A) $y' = t^2$~~

~~B) $y' = \frac{1}{2}t + 1$~~

~~C) $y' = e^t$~~

~~D) $y' = t + 1$~~

~~E) $y' = -2e^t$~~

F) $y' = y - t$

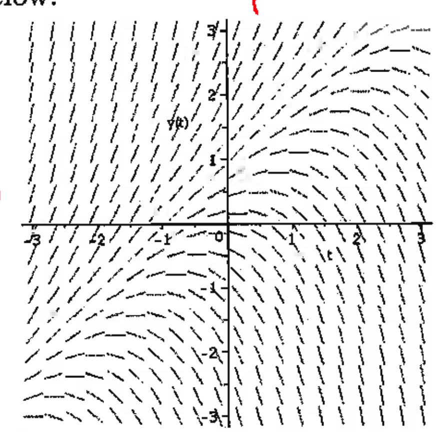
~~G) $y' = ln(t)$~~

~~H) $y' = 0$~~

~~I) $y' = sin(t)$~~

~~J) $y' = cos(t)$~~

$y' = f(t, y)$



IVP: Does C has unique soln when plugging in initial value $y(t_0) = y_0$

Examples: No solution:

Ex 1: $y' = y' + 1$

Ex 2: $(y')^2 = -1$

Ex 3 (IVP): $\frac{dy}{dx} = y(1 + \frac{1}{x})$, $y(0) = 1$

$\int \frac{dy}{y} = \int (1 + \frac{1}{x}) dx$ implies $ln|y| = x + ln|x| + C$

$|y| = e^{x + ln|x| + C} = e^x e^{ln|x|} e^C = C|x|e^x = Cxe^x$

$y = \pm Cxe^x$ implies $y = Cxe^x$

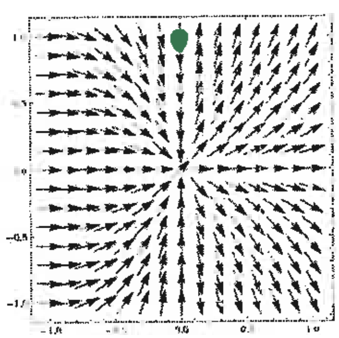
$y(0) = 1$: $1 = C(0)e^0 = 0$ implies

IVP $\frac{dy}{dx} = y(1 + \frac{1}{x})$, $y(0) = 1$ has no solution.

to IVP

http://www.wolframalpha.com

slope field: $\{1, y(1+1/x)\}/sqrt(1+y\wedge 2(1+1/x)\wedge 2)$



Special cases:

Calc 1

Suppose f is cont. on (a, b) and the point $t_0 \in (a, b)$,
Solve IVP: $\frac{dy}{dt} = f(t)$, $y(t_0) = y_0$

$$dy = f(t)dt$$

$$\int dy = \int f(t)dt$$

$y = F(t) + C$ where F is any anti-derivative of F.

Initial Value Problem (IVP): $y(t_0) = y_0$

$$y_0 = F(t_0) + C \text{ implies } C = y_0 - F(t_0)$$

Hence unique solution (if domain connected) to IVP:

$$y = F(t) + y_0 - F(t_0)$$

First order linear differential equation:

Thm 2.4.1: If p and g are continuous on (a, b) and the point $t_0 \in (a, b)$, then there exists a unique function $y = \phi(t)$ defined on (a, b) that satisfies the following initial value problem:

$$y' + p(t)y = g(t), \quad y(t_0) = y_0.$$

More general case (but still need hypothesis)

Thm 2.4.2: Suppose the functions $z = f(t, y)$ and $\boxed{z = \frac{\partial f}{\partial y}(t, y)}$ are continuous on $(a, b) \times (c, d)$ and the point $(t_0, y_0) \in (a, b) \times (c, d)$,

then there exists an interval $(t_0 - h, t_0 + h) \subset (a, b)$ such that there exists a unique function $y = \phi(t)$ defined on $(t_0 - h, t_0 + h)$ that satisfies the following initial value problem:

$$y' = f(t, y), \quad y(t_0) = y_0.$$

cont

If possible **without solving**, determine where the solution exists for the following initial value problems:

If not possible **without solving**, state where in the ty-plane, the hypothesis of theorem 2.4.2 is satisfied. In other words, use theorm 2.4.2 to determine where for some interval about t_0, a solution to IVP, $y' = f(t, y)$, $y(t_0) = y_0$ exists and is unique.

Example 1: $ty' - y = 1, \ y(t_0) = y_0$

Example 2: $y' = ln|\frac{t}{y}|, \ y(3) = 6$

Example 3: $(t^2 - 1)y' - \frac{t^3 y}{t-4} = ln|t|, \ y(3) = 6$

Section 2.4 example: $\frac{dy}{dt} = \frac{1}{(1-t)(2-y)}$

$F(y, t) = \frac{1}{(1-t)(2-y)}$ is continuous for all $t \neq 1, \ y \neq 2$

$$\frac{\partial F}{\partial y} = \frac{\partial\left(\frac{1}{(1-t)(2-y)}\right)}{\partial y} = \frac{1}{(1-t)} \frac{\partial (2-y)^{-1}}{\partial y} = \frac{1}{(1-t)(2-y)^2}$$

$\frac{\partial F}{\partial y}$ is continuous for all $t \neq 1, \ y \neq 2$

Thus the IVP $\frac{dy}{dt} = \frac{1}{(1-t)(2-y)}$, $y(t_0) = y_0$ has a unique solution if $t_0 \neq 1, \ y_0 \neq 2$.

Note that if $y_0 = 2$, $\frac{dy}{dt} = \frac{1}{(1-t)(2-y)}$, $y(t_0) = 2$ has two solutions if $t_0 \neq 1$ (and if we allow vertical slope in domain. Note normally our convention will be to NOT allow vertical slope in domain of solution).

Note that if $t_0 = 1$, $\frac{dy}{dt} = \frac{1}{(1-t)(2-y)}$, $y(1) = y_0$ has no solutions.