1.) If g is continuous at $t = 0$, then there is a unique solution to the differential equation $ay'' + by + cy = g(t)$, $y(0) = 1$, $y'(0) = 3$

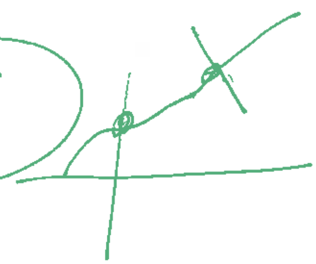
A) True **B) False**

2.) If g is continuous, then there is a unique solution to the differential equation $ay'' + by + cy = g(t)$, $y(0) = 1$, $y(1) = 3$

A) True **B) False**

3.) If g is continuous, then there is a unique solution to the differential equation $ay'' + by + cy = g(t)$, $y(0) = 1$, $y'(1) = 3$

A) True **B) False**



4.) If g is continuous, then there is a unique solution to the differential equation $ay'' + by + cy = g(t)$, $y(1) = 1$, $y'(1) = 3$

A) True **B) False**

3.3: Linear Independence and the Wronskian

Defn: f and g are linearly dependent if there exists constants c_1, c_2 such that $c_1 \neq 0$ or $c_2 \neq 0$ and

$$c_1 f(t) + c_2 g(t) = 0 \text{ for all } t \in (a, b)$$

Thm 3.3.1: If $f : (a, b) \to R$ and $g(a, b) \to R$ are differentiable functions on (a, b) and if $W(f, g)(t_0) \neq 0$ for some $t_0 \in (a, b)$, then f and g are linearly independent on (a, b). Moreover, if f and g are linearly dependent on (a, b), then $W(f, g)(t) = 0$ for all $t \in (a, b)$

If $c_1 f(t) + c_2 g(t) = 0$ for all t, then $c_1 f'(t) + c_2 g'(t) = 0$

Solve the following linear system of equations for c_1, c_2

$$c_1 f(t_0) + c_2 g(t_0) = 0$$
$$c_1 f'(t_0) + c_2 g'(t_0) = 0$$

$$\begin{bmatrix} f(t_0) & g(t_0) \\ f'(t_0) & g'(t_0) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

3.5 General sol to nonhom linea DE

$y = c_1\phi_1 + c_2\phi_2 + \psi$

Thm: Suppose $c_1\phi_1(t) + c_2\phi_2(t)$ is a general solution to

$$ay'' + by' + cy = 0,$$

If ψ is a solution to

$$ay'' + by' + cy = g(t) \text{ [*]},$$

claim

$\boxed{\text{Then } \psi + c_1\phi_1(t) + c_2\phi_2(t) \text{ is also a solution to [*].}}$

Moreover if γ is also a solution to [*], then there exist constants c_1, c_2 such that

$$\gamma = \psi + c_1\phi_1(t) + c_2\phi_2(t)$$

Or in other words, $\psi + c_1\phi_1(t) + c_2\phi_2(t)$ is a general solution to [*].

Proof:

Define $L(f) = af'' + bf' + cf$.

Recall L is a linear function.

Let $h = c_1\phi_1(t) + c_2\phi_2(t)$. Since h is a solution to the differential equation, $ay'' + by' + cy = 0$,

$$ah'' + bh' + ch = 0$$

Since ψ is a solution to $ay'' + by' + cy = g(t)$,

$$a\psi'' + b\psi' + c\psi = g(t)$$

We will now show that $\psi + \underbrace{c_1\phi_1(t) + c_2\phi_2(t)}_{h} = \psi + h$ is also a solution to [*].

$a(\psi+h)'' + b(\psi+h)' + c(\psi+h) =$

$a\psi'' + ah'' + b\psi' + bh' + c\psi + ch$

$= (a\psi'' + b\psi' + c\psi) + (ah'' + bh' + ch) = g + 0 = g(t)$

Since γ a solution to $ay'' + by' + cy = g(t)$,

$a\gamma'' + b\gamma' + c\gamma = g(t)$

We will first show that $\gamma - \psi$ is a solution to the differential equation $ay'' + by' + cy = 0$.

$a(\gamma - \psi)'' + b(\gamma - \psi)' + c(\gamma - \psi)$

$a\gamma'' + b\gamma' + c\gamma - (a\psi'' + b\psi' + c\psi)$

$= g(t) - g(t) = 0$

Since $\gamma - \psi$ is a solution to $ay'' + by' + cy = 0$ and

$c_1\phi_1(t) + c_2\phi_2(t)$ is a general solution to
$$ay'' + by' + cy = 0,$$

there exist constants c_1, c_2 such that

$$\gamma - \psi = \underline{c_1\phi_1 + c_2\phi_2}$$

Thus $\gamma = \psi + c_1\phi_1(t) + c_2\phi_2(t)$.

General solution to 2^{nd} order linear nonhog eqn

$y(t) = c_1\phi_1 + c_2\phi_2 + \psi$

Thm:
Suppose f_1 is a a solution to $ay'' + by' + cy = g_1(t)$ and f_2 is a a solution to $ay'' + by' + cy = g_2(t)$, then $f_1 + f_2$ is a solution to $ay'' + by' + cy = g_1(t) + g_2(t)$

Proof: Let $L(f) = af'' + bf' + cf$.

Since f_1 is a solution to $ay'' + by' + cy = g_1(t)$,

Since f_2 is a solution to $ay'' + by' + cy = g_2(t)$,

We will now show that $f_1 + f_2$ is a solution to $ay'' + by' + cy = g_1(t) + g_2(t)$.

Sidenote: The proofs above work even if a, b, c are functions of t instead of constants.