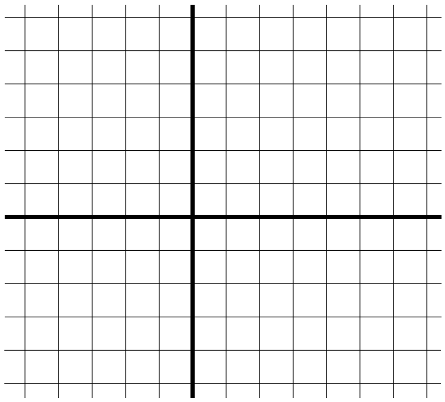
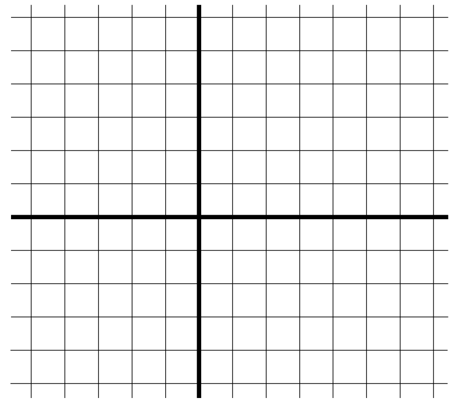
Math 3600 Differential Equations Exam #1
Sept 26, 2018 SHOW ALL WORK

[10] 1a.) Draw the direction field for the following differential equation: $y' = y(2 - y)$





[4] 1b.) On the direction field above, draw the solution to the above differential equation that satisfies the initial condition $y(0) = 1$.

[6] 1c.) Does the differential equation whose direction field is given above have any equilibrium solutions? If so, state whether they are stable, semi-stable or unstable.

[5] 2.) Give an example of an initial value problem that does not have a unique solution.

3.) Circle T for true and F for false.

[5] 2c.) Suppose $y = \phi_1(t)$ and $y = \phi_2(t)$ are solutions to $ay'' + by' + cy = 0$. Then $y = c_1\phi_1(t) + c_2\phi_2(t)$ is also a solution to this linear homogeneous differential equation.

T F

[5] 2d.) Suppose $y = \phi_1(t)$ and $y = \phi_2(t)$ are linearly independent solutions to $ay'' + by' + cy = 0$. If $y = h(t)$ is also a solution to $ay'' + by' + cy = 0$, then there exists constants c_1 and c_2 such that $h(t) = c_1\phi_1(t) + c_2\phi_2(t)$.

T F

[20] 4.) Find the general solution to $ty' - 2y = t^3e^t - 8$. Also find the solution that passes thru the point (1, 3). How does the solution passing thru (1, 3) behave as $t \to \infty$?

General solution: ______________________________

IVP solution: ______________________________

$t \to \infty$, $y \to$ ______________________________

5.) Solve the following 2nd order differential equations

[15] 5a.) $2y'' - y' + 10y = 0$

General solution: ____________________

[15] 5b.) $(x)(x'') = (x')^2$ Hint: Let $x' = \frac{dx}{dt} = v$, then $v' =$

General solution: ____________________

[15] 6.) Show by induction that for Picard's iteration method, $\phi_n(t) = \sum_{k=1}^{n} \frac{t^{2k}}{k!}$ approximates the solution to the initial value problem, $y' = 2t(1+y)$, $y(0) = 0$ where $\phi_1(t) = t^2$. You may use the proof outline below or write it from scratch.

Proof by induction on n.

For $n = 1$, $\sum_{k=1}^{1} \frac{t^{2k}}{k!} =$

Suppose for $n = j$, $\phi_{j-1}(t) = \sum_{k=1}^{j-1} \frac{t^{2k}}{k!}$

Claim: $\phi_j =$

Proof of claim: By Picard's iteration method, $\phi_j =$