

Let $\tilde{X} = P\tilde{y}$ for change of basis

$$\tilde{X}' = A \tilde{x}$$

$$[P\tilde{y}]' = A P\tilde{y}$$

$$\text{Thus } \lambda = \frac{2a \pm \sqrt{4a^2 - 4(a^2 + b^2)}}{2} = \frac{2a \pm \sqrt{-4b^2}}{2} = a \pm bi$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ implies } \begin{cases} x'_1 = ax_1 + bx_2 \\ x'_2 = -bx_1 + ax_2 \end{cases}$$

$$\text{Change to polar coordinates: } r^2 = x_1^2 + x_2^2 \text{ and } \tan\theta = \frac{x_2}{x_1}$$

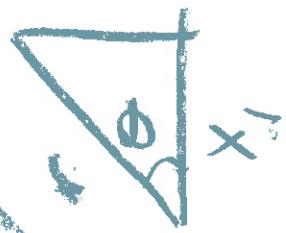
Take derivative with respect to t of both equations:

$$2rr' = 2x_1x'_1 + 2x_2x'_2 \text{ implies}$$

$$rr' = x_1(ax_1 + bx_2) + x_2(-bx_1 + ax_2)$$

$$= ax_1^2 + bx_1x_2 - bx_1x_2 + ax_2^2 = a(x_1^2 + x_2^2) = ar^2$$

$$\text{Thus } rr' = ar^2 \text{ implies } \frac{dr}{dt} = ar \text{ and thus } r = Ce^{at}.$$



$$(\sec^2\theta)\theta' = \frac{x_1x'_2 - x'_1x_2}{x_1^2} = \frac{x_1(-bx_1 + ax_2) - (ax_1 + bx_2)x_2}{x_1^2} = \frac{-b(x_1^2 + x_2^2)}{x_1^2} = -b\sec^2\theta$$

$$(\sec^2\theta)\theta' = -b\sec^2\theta \text{ implies } \theta' = -b \text{ and thus } \theta = -bt + \theta_0$$