22M:100 (MATH:3600:0001) Exam 2
April 20, 2016 **Show all work**

[10] 1.) Find the radius of convergence of the power series $\sum_{n=2}^{\infty} \frac{(-2)^n(x-4)^n}{n^2}$

2.) Circle T for True and F for false.

[3] 2a.) Suppose $f(x) = \Sigma a_n(x-4)^n$ has a radius of convergence $= r$ about the point 4. Then we can define the domain of f to be $(4-r, 4+r)$.

T F

[3] 2b.) Suppose $f(x) = \Sigma a_n(x-4)^n$ has a radius of convergence $= r$ about the point 4. Then we can define the domain of f to be $(r-4, r+4)$.

T F

[3] 2c.) The radius of convergence of the power series for $f(x) = \frac{x}{(x^2+9)(x+5)}$ about the point $x_0 = 2$ is at least as large as $\sqrt{13}$.

T F

[3] 2d.) Let $f(x) = \frac{x}{(x^2+9)(x+5)}$. Then $\frac{x}{(x^2+9)(x+5)} = \Sigma_{n=0}^{\infty} a_n(x-2)^n$ where $a_n = \frac{f^{(n)}(2)}{n!}$ for all values of $x \in (2-\sqrt{13},\ 2+\sqrt{13})$.

T F

3.) Given the differential equation $2xy'' - (1+x)y' + y = 0$,

[5] i.) Determine if $x = 0$ is an ordinary point, regular singular point or irregular singular point.

[15] ii.) Determine the indicial equation, the roots of the indicial equation, and the recurrence relation.

indicial equation: ______________________________ roots of the indicial equation: ____________

recurrence relation: ____

[5] 4i.) Show that $\mathbf{x} = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} t + \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix} \right] e^t$ is a solution to the differential equation $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \mathbf{x}$

[5] 4ii) Find a second solution to $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \mathbf{x}$

[5] 4iii) State the general solution to $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \mathbf{x}$

[5] 4iv) Solve the initial value problem $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \mathbf{x}$, $\mathbf{x}(0) = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

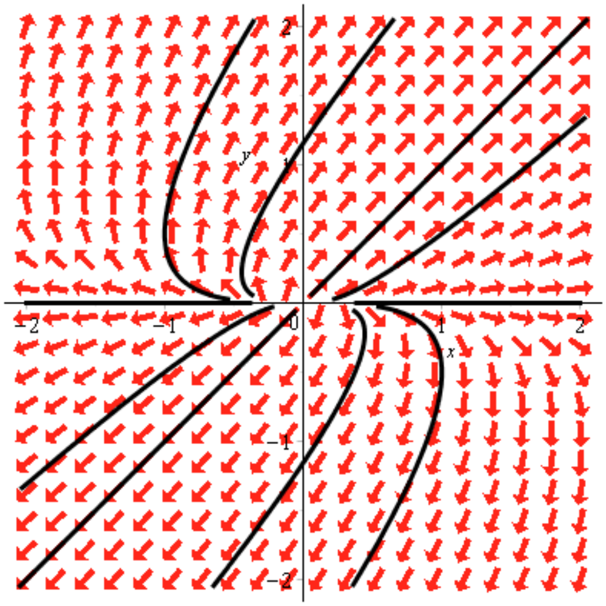

Fig. 1. Phase portrait

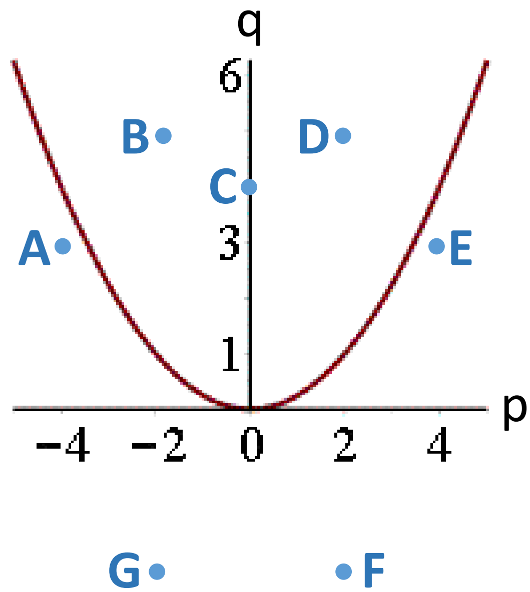

Fig 2. Stability diagram

5.) The phase portrait for a differential equation (not given) is shown above in Fig 1. Answer the following questions about this differential equation and its solution.

[9] i.) Find all equilibrium solutions and determine whether the critical point is asymptotically stable, stable, or unstable. Also classify it as to type (nodal source, nodal sink, saddle point, spiral source, spiral sink, center).

Equilibrium solution: ________________________________

Stability : ________________________________ Type: ________________________________

[6] ii.) Which of the differential equations below matches the phase portrait shown above?

a.) $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}\mathbf{x}$ b.) $\mathbf{x}' = \begin{pmatrix} -1 & 2 \\ 0 & -3 \end{pmatrix}\mathbf{x}$ c.) $\mathbf{x}' = \begin{pmatrix} -1 & 2 \\ 0 & 3 \end{pmatrix}\mathbf{x}$ d.) $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}\mathbf{x}$

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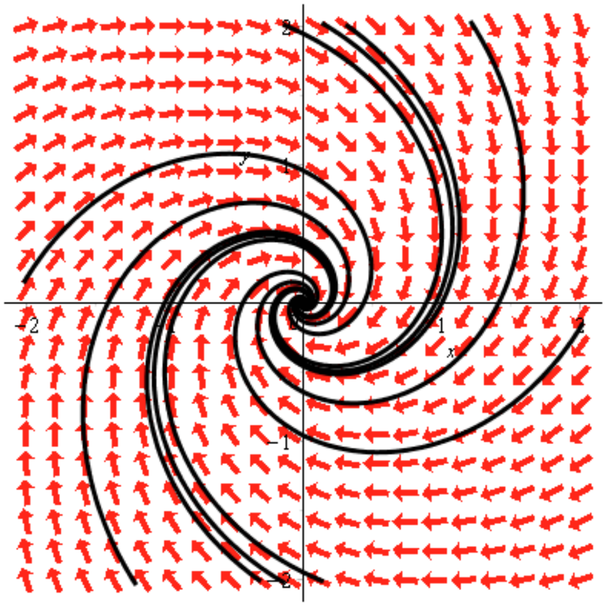
e.) $\mathbf{x}' = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}\mathbf{x}$ f.) $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}\mathbf{x}$ g.) $\mathbf{x}' = \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix}\mathbf{x}$ h.) $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\mathbf{x}$

[4] iii.) The (p, q) value corresponding to this differential equation is plotted in the Fig 2 graph above. Circle the letter corresponding to the (p, q) value corresponding to the differential equation, $\mathbf{x}' = A\mathbf{x}$ whose phase portrait is drawn above. Recall that the eigenvalues of A are $\frac{p \pm \sqrt{p^2 - 4q}}{2}$

A **B** **C** **D** **E** **F** **G**



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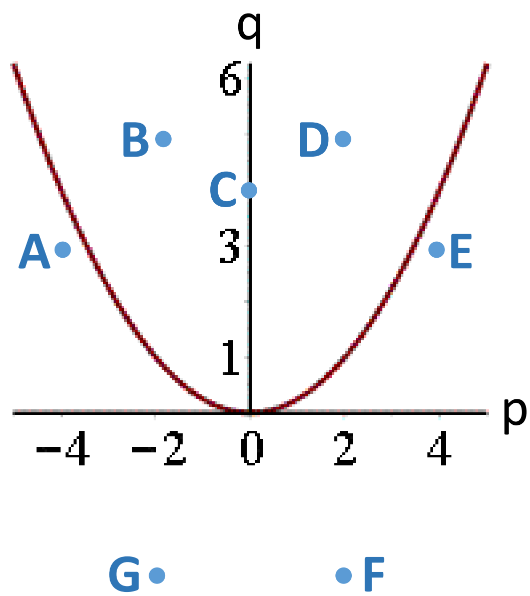

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