

## M133 Exam 1

Choose 3 of the following 6 problems. Clearly indicate your choice. You may do more than 3 problems. If you do more than 3 problems, I may change which 3 problems count toward your grade on the in class part of exam 1 if this improves your grade.

1.) Describe a pre-atlas for  $RP^1$  which generalizes to  $RP^n$ . Briefly describe how you can generalize your pre-atlas for  $RP^1$  to  $RP^n$

2.) Show that  $f : RP^1 \rightarrow S^1$ ,  $f([(cos(\theta), sin(\theta))]) = (cos(2\theta), sin(2\theta))$  is smooth (note: trig functions and inverse trig functions are smooth).

3.) Suppose  $f : M \rightarrow N$  is smooth where  $M$  is an  $m$ -manifold and  $N$  is an  $n$ -manifold. Show that  $f$  is a continuous.

4.) We say that two pre-atlases on an  $m$ -manifold are *equivalent* if their union is an atlas. Show that this is an equivalence relation and that each equivalence class contains a unique complete atlas.

5a.) Define Lie group.

5b.) Show that  $(\mathbf{Z}, +)$  is a Lie group.

6.) Define  $f(S^1) \rightarrow S^1$ ,  $f(e^{i\theta}) = e^{2i\theta}$ . Calculate  $df_{(1,0)}$ .