n-Permutation of n objects: $P(n,n) = n! =$ number of ways to place n nonattacking rooks on an $n \times n$ chessboard.

r-Permutation of n objects: $P(n,r) = n(n-1)...(n-r+1) =$ number of ways to place r nonattacking rooks on an $r \times n$ chessboard.

$C(n,r)P(n,r) = n(n-1)...(n-r+1) =$ number of ways to place r nonattacking rooks on an $n \times n$ chessboard.

Inclusion/Exclusion:

6.1/6.5: n-Permutation of n objects: use to exclude a pattern anywhere from within the permutation

6.4: n-Permutation of n objects with forbidden positions $a_i \notin X_i$ (ex: Derangements: $a_i \neq i$)

can also use Inclusion/Exclusion to find r-Permutation of n objects or permutations of multisets.

3.4 Permutations of Multisets

Thm 3.4.1: Let $A = \{\infty \cdot 1, \infty \cdot 2, ..., \infty \cdot k\}$

The number of r permutations of $A = k^r$.

Thm 3.4.2: Let $B = \{n_1 \cdot 1, n_2 \cdot 2, ..., n_k \cdot k\}$

$n = n_1 + n_2 + ... + n_k$

The number of n-permutations of $B = \frac{n!}{n_1! n_2! \cdots n_k!}$.

If want r-permutations of B, need to use or statements or technique from later chapter.

7.7: coefficient of $\frac{x^n}{n!}$ = the number of permutations of
$B = \{n_1 \cdot 1, n_2 \cdot 2, ..., n_k \cdot k\}$ ($N_i \in \mathcal{Z}_{\rangle} \cup \infty$) satisfying properties P_i.

Ex: the number of r-permutations of $\{3 \cdot 1, \infty \cdot 2\}$ where there are an odd number of 1's and an even number of 2's is 0 if r even and $\frac{(2n+1)!}{(2n)!} + \frac{(2n+1)!}{3!(2n-2)!}$ if $r = 2n+1$.

$$(x+\frac{x^3}{3!})(1+\frac{x^2}{2!}+\frac{x^4}{4!}+...) = x+(\frac{3!}{2!}+\frac{3!}{3!})\frac{x^3}{3!}+(\frac{5!}{4!}+\frac{5!}{3!2!})\frac{x^5}{5!}+...+(\frac{(2n+1)!}{(2n)!}+\frac{(2n+1)!}{3!(2n-2)!})\frac{x^{2n+1}}{(2n+1)!}+...$$

$e^{ax} = \Sigma_{n=0}^{\infty} \frac{(ax)^n}{n!}$. Thus $\frac{e^x+e^{-x}}{2} = \Sigma_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$ and $\frac{e^x-e^{-x}}{2} = \Sigma_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$