Determine the generating function for h_n = the number of ways to make n cents using pennies, nickels, dimes, and quarters.

Note h_n = the number of nonnegative integral solutions to

$$e_1 + 5e_2 + 10e_3 + 25e_4 = n$$

Let $f_1 = e_1,\ f_2 = 5e_2,\ f_3 = 10e_3,\ f_4 = 25e_4,$

Then h_n = the number of nonnegative integral solutions to $f_1 + f_2 + f_3 + f_4 = n$

where f_1 is a nonnegative integer

f_2 is a multiple of 5

f_3 is a multiple of 10

f_4 is a multiple of 25

Hence the generating function for h_n is

HW: ch 7: 30ad, 34, 35

5.6

$\frac{1}{(1-z)^n} = \Sigma_{k=0}^{\infty} \binom{-n}{k} (-z)^k = \Sigma_{k=0}^{\infty} \binom{n+k-1}{k} z^k$, $|z| < 1$

Thus $\frac{1}{(1-rx)^n} = \Sigma_{k=0}^{\infty} \binom{n+k-1}{k} r^k x^k$ if $|x| < \frac{1}{|r|}$

7.5 Recurrences and Generating Functions.

Use a generating function to solve the recurrence relation:

$h_n = 2h_{n-1} + 8h_{n-2}$, $h_0 = -2$, $h_1 = 22$.