## 2.1 Basic Counting

A partition of a set S is a collection of subsets  $S_i$  of S such that  $S = \bigcup S_i$  and  $S_i \cap S_j = \emptyset$  for all  $i \neq j$ .

Addition Principle: If  $S = S_1 \cup S_2$  and  $S_1 \cap S_2 = \emptyset$ , then  $|S| = |S_1| + |S_2|$ .

If  $S_1 \cap S_2 = \emptyset$  and if  $x \in S$  implies  $x \in S_1$  OR  $x \in S_2$ , then  $|S| = |S_1| + |S_2|$ .

Multiplication Principle: If  $S = S_1 \times S_2$ , then  $|S| = |S_1||S_2|$ .  $x = (a, b) \in S \text{ implies } a \in S_1 \text{ AND } b \in S_2, \text{ then } |S| = |S_1||S_2|.$ 

**Subtraction Principle:** Suppose  $A \subset U$ . Let the complement of A in  $U = \overline{A} = \{x \in U \mid x \notin A\}$ . Then  $|A| = |U| - |\overline{A}|$ .

**Division Principle:** Suppose  $S = \bigcup_{i=1}^k S_i$ . If  $|S_i| = n \ \forall i$ , then  $k = \frac{|S|}{n}$ .

# Counting Problems:

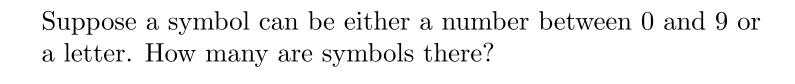
- 1.) Order matters (ordered arrangements or selections)
  - 1a.) no repeats allowed
  - 1b.) (limited) repeats allowed
- 2.) Order does not matter (unordered arrangements or selections)
  - 2a.) no repeats allowed
  - 2b.) (limited) repeats allowed

Defn: A *multiset* is a collection of objects were repeats are allowed.

Set: 
$$\{a, a, b, b, b\} = \{a, b\}$$

Multiset: 
$$\{a, a, b, b, b\} = \{2 \cdot a, 3 \cdot b\}$$

Subsets: Suppose a set B has n elements (i.e., |B| = n). The number of subsets of B is



How many sequences consisting of one letter followed by one single digit number (0 - 9) are possible?

How many different license plates are possible if 3 letters followed by 3 numbers are used?

How many different license plates are possible if 3 letters followed by 3 numbers are used and the license plate starts with a vowel if and only if the plate contains exactly one vowel?

### Subsets

Suppose a set A has four elements (i.e., the cardinality of A = |A| = 4)

The number of subsets of A is

The number of nonempty subsets of A is

A pizza parlor offers 4 different toppings (sausage, onions, chicken, walnuts). How many different types of pizzas can one order?

Suppose a set B has n elements (i.e., |B| = n). The number of subsets of B is

Example: How many 10-digit telephone numbers are there if 1.) there are no restrictions.
2.) the digits must all be distinct.
3.) The area code cannot begin with a 0 or 1 and must have a 0 or 1 in the middle.
Example: How many different seven-digit numbers can be constructed out of the digits 2, 4, 8, 8, 8, 8, 8?
Example: How many different seven-digit numbers can be constructed out of the digits 2, 2, 8, 8, 8, 8, 8?

Example A: How many numbers between 100 and 1000 have distinct digits.	ve
Example B: How many odd numbers between 100 and 100 have distinct digits.	)0
Example C: How many even numbers between 100 and 100 have distinct digits.  method 1:	)0
method 2:	

### 2.2 Permutations:

Suppose |S| = n.

An r-permutation of S is an ordered arrangement of r of the n elements of S.

If r = n, then an r-permutation of S is a permutation of S.

P(n,r) = number of r-permutations of S where |S| = n.

4 TA's need to be assigned to 4 different classes. How many different possible assignments are there?

4 classes need to be assigned a TA. There are 10 TAs. How many different possible assignments are there?

If r > n, then P(n, r) =

$$P(0,0) = P(n,0) = P(n,1) = P(n,n) =$$

$$n! = n(n-1)(n-2)...(2)(1)$$

0! = 1

Thm 2.2.1: If  $r \le n$ , then  $P(n,r) = \frac{n!}{(n-r)!}$ 

#### 2.3 Combinations

An r-combination of S is an r-element subset of S (ORDER DOES NOT MATTER).

C(n,r) = number of r-combinations of S where |S| = n.

How many different math teams consisting of 4 people can be formed if there are 10 students from which to choose?

Thm: 
$$C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!}$$

Cor: 
$$C(n,r) = C(n,n-r)$$

Cor: 
$$C(n,r) = C(n-1,r-1) + C(n-1,r)$$

Cor: Pascal's Triangle.

Cor: 
$$\sum_{i=0}^{n} \binom{n}{i} = 2^n$$

How many different proteins containing 10 amino acids can be formed if the protein contains 5 alanines(A), 3 leucines (L), and 2 serines (S)?