3.3 Combinations

$C(n,r) = \begin{pmatrix} n \\ r \end{pmatrix} = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!}$

$\Sigma_{i=0}^{n} \begin{pmatrix} n \\ i \end{pmatrix} = 2^n$

3.4 Permutations of Multisets

Thm 3.4.1: Let $A = \{\infty \cdot 1, \infty \cdot 2, ..., \infty \cdot k\}$

The number of r permutations of $A = k^r$.

Ex: The number of 8-digit numbers with digits $\{1,2,3,4\} = 4^8$

Ex: The number of 9-digit numbers with digits $\{0,1,2,3\} = 3(4^8)$

Thm 3.4.2: Let $B = \{n_1 \cdot 1, n_2 \cdot 2, ..., n_k \cdot k\}$

$n = n_1 + n_2 + ... + n_k$

The number of permutations of $B = \frac{n!}{n_1!n_2!\cdots n_k!}$.

$\frac{n!}{n_1!n_2!\cdots n_k!} = \begin{pmatrix} n \\ n_1 \end{pmatrix} \begin{pmatrix} n-n_1 \\ n_2 \end{pmatrix} \cdots \begin{pmatrix} n - \Sigma_{i=1}^{k-1} n_i \\ n_k \end{pmatrix}$.

# of permutations of $\{n_1 \cdot 1, (n-n_1) \cdot 2\} = \frac{n!}{n_1!(n-n_1)!} = C(n, n_1)$

Thm 3.4.3 If have $n = n_1 + n_2 + ... + n_k$ different objects to be placed in k labeled boxes such that the box B_i contains n_i objects

3.4 Combinations of Multisets

Let $A = \{\infty \cdot 1, \infty \cdot 2, ..., \infty \cdot k\}$

The number of r combinations of $A = \binom{r+k-1}{r}$

Proof: An r combinations is