www.geometrygames.org/TorusGames

Equivalence class $[a] = \{x \mid x \sim a\}$

$\mathcal{P} = \{P_\alpha \mid \alpha \in A\}$ is a partition of X iff
$X = \cup_{P_\alpha \in \mathcal{P}} P_\alpha$, $P_\alpha \neq \emptyset \ \forall \alpha$, and $P_\alpha \cap P_\beta \neq \emptyset$ implies $P_\alpha = P_\beta$

Ex: Suppose $a, b \in \mathcal{Z}$. $a \sim b$ if $ab > 0$

$[4] =$

$[-2] =$

$[0] =$

Ex: $\mathcal{Z} =$

Thm 4.5.3: If $\sim$ is an equivalence relation on X, then
$\{[x_\alpha] \mid x_\alpha \in X\}$ is a partition of X.

If $\mathcal{P} = \{P_\alpha \mid \alpha \in A\}$ is a partition of X, then
$x \sim y$ iff $\exists P_\alpha$ such that $x, y \in P_\alpha$ is an equivalence relation.

Proof: Suppose $\sim$ is an equivalence relation on X.

Claim: $\{[x_\alpha] \mid x_\alpha \in X\}$ is a partition of X.

Let $x_\alpha \in X$. Then $x_\alpha \in [x_\alpha]$ since $\sim$ is reflexive.
Thus $[x_\alpha] \neq \emptyset$ and $X = \cup_{x_a \in X} [x_\alpha]$.

Suppose $[x_\alpha] \cap [x_\beta] \neq \emptyset$.

Claim: $[x_\alpha] = [x_\beta]$

Claim: $[x_\alpha] \subset [x_\beta]$ and $[x_\beta] \subset [x_\alpha]$

Claim: If $z \in [x_\alpha]$, then $z \in [x_\beta]$ (and similarly for the other inclusion).

Proof of Claim: Since $z \in [x_\alpha]$, $z \sim x_\alpha$.

Suppose $\mathcal{P} = \{P_\alpha \mid a \in A\}$.

Claim: $x \sim y$ iff there exists $P_\alpha \in \mathcal{P}$ such that $x, y \in P_\alpha$ is an equivalence relation on X.

Proof of Claim: HW #44 (don't assume finite).