HW 3

Chapter 3:

50) Place two of the rooks in two non attacking positions.

Choose two of 8 columns: $C(8,2) = \frac{(8!)}{(6!)(2!)}$

Note when choosing the two columns, the order does NOT matter.

Place one rook in these 2 columns in two different rows: $P(8,2) = \frac{(8!)}{(6!)}$

Note when choosing the two rows, the order DOES matter. For example, suppose columns a and b were first chosen. If rows c and d are chosen in the order c, d, then the rooks are placed in squares $(a,c), (b,d)$ while if the rows c and d are chosen in the order d, c, then the rooks are placed in squares $(a,d), (b,c)$

Place two more rooks to form a rectangle: 1 choice

Place the fifth rook: 64 - 4 = 60 choices

Place two of the rooks in two non attacking positions AND place two more rooks to form a rectangle AND place the fifth rook: $\frac{(8!)}{(6!)(2!)}\frac{(8!)}{(6!)}(60)$

51) Number of n-combinations from the multiset $\{n \cdot a, 1, 2, ..., n\}$

Choose k elements from $S = \{1, 2, ..., n\}$: $C(n,k)$

Choose $n-k$ elements from $T = \{n \cdot a\}$: 1

Choose k elements from $\{1, 2, ..., n\}$ AND choose $n-k$ elements from $\{n \cdot a\}$: $C(n,k)$

Choose (0 elements from S AND n elements from T) OR (1 element from S AND $n-1$ elements from T) OR (2 elements from S AND $n-2$ elements from T) OR ... OR (n elements from S AND 0 elements from T):

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + ... + \binom{n}{n} = 2^n$$

52) Number of n-combinations from the multiset $\{n \cdot a, n \cdot b, 1, 2, ..., n+1\}$

Choose k elements from $S = \{1, 2, ..., n+1\}$: $C(n+1,k)$

Choose a number j such that $0 \le j \le n-k$: $n-k+1$:

Choose j elements from T and $n-k-j$ elements from R: 1

Choose k elements from S AND choose $n-k$ elements from $T \cup R$: $(n-k+1)C(n+1,k) = (n+1)C(n,k)$

Choose (0 elements from S AND n elements from $T \cup R$) OR (1 element from S AND $n-1$ elements from $T \cup R$) OR (2 elements from S AND $n-2$ elements from $T \cup R$) OR ... OR (n elements from S AND 0 elements from $T \cup R$):

$$(n+1)[\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + ... + \binom{n}{n}] = (n+1)2^n$$