5.5) Integration by substitution

If $F(x) = ln|x^2 - x|$, then by chain rule $F'(x) = \frac{1}{x^2-x}(2x-1) = \frac{2x-1}{x^2-x}$

Thus $\int \frac{2x-1}{x^2-x}dx =$

How do you recognize derivative when chain rule involved? u - substition

$\int \frac{2x-1}{x^2-x}dx = \int \frac{du}{u} = \int \frac{1}{u}du = ln|u| + C = ln|x^2 - x| + C$

Let $u = x^2 - x$, then $du = 2x - 1$

5.5 Examples.

1.) $\int 2xe^{x^2} dx$

2.) $\int 3x^2\sqrt{x^3 + 1}dx$

3.) $\int \frac{xdx}{x^2+4}$

4.) $\int x\sqrt{1 + x}dx$

5.) $\int \sqrt{x}(x^2 - 1)dx$

6.) $\int cos^3(x)dx$

7.) $\int_0^\pi cos^3(x)dx = \int_0^\pi cos^2(x)cos(x)dx = \int_0^\pi (1-sin^2(x))cos(x)dx$
$= \int_0^0 (1-u^2)du = 0$

Let $u = sin(x)$, $du = cos(x)dx$,
when $x = 0$, $u = sin(0) = 0$, when $x = \pi$, $u = sin(\pi) = 0$

Shortcut method: Use symmetry.

For example:
If f is an odd function $(f(-x) = -f(x))$, then $\int_{-a}^a f(x)dx = 0$
If f is an even function, then $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$