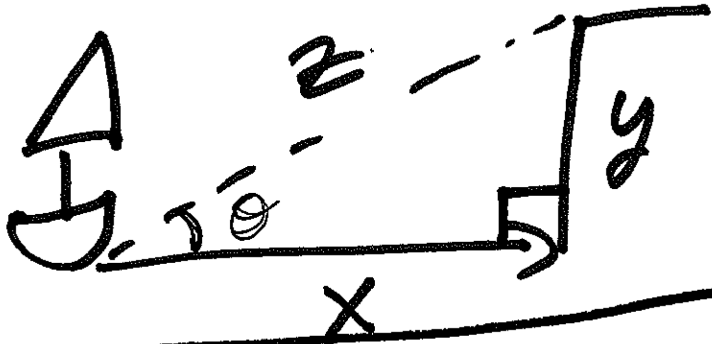


$x^2+y^2=z^2$

constant

Is $\frac{dx}{dt}$ increasing?

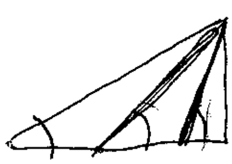
yes (to ∞)

$$\cancel{2}x\frac{dx}{dt}+0=\cancel{2}z\frac{dz}{dt}$$

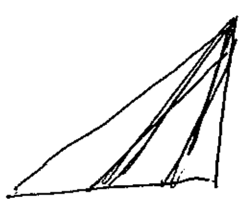
constant

$$\frac{dx}{dt}=\frac{z}{x}\frac{dz}{dt}$$

note $x \to 0$ $\Rightarrow \frac{dx}{dt} \to \infty$



$$=\frac{1}{\cos\theta}\frac{dz}{dt} \to \infty$$



$\theta \to 90^\circ = \frac{\pi}{2} \Rightarrow \cos\theta \to 0$

$$\left(\frac{\sin x}{e^x}\right)' = \frac{e^x \cos x - (\sin x) e^x}{e^{2x}}$$

$$= \frac{\cos x - \sin x}{e^x}$$

$$y = \frac{\sin x}{e^x}$$

$$\ln y = \ln\left(\frac{\sin x}{e^x}\right)$$

$$= \ln(\sin x) - \ln(e^x)$$

$$\ln y = \ln(\sin x) - x$$

$$\frac{y'}{y} = \frac{\cos x}{\sin(x)} - 1$$

$$y' = y\left(\frac{\cos x}{\sin x} - 1\right)$$

$$= \frac{\sin x}{e^x}\left(\frac{\cos x}{\sin x} - 1\right)$$

$$= \frac{\cos x - \sin x}{e^x}$$

$$(e^x)' = e^x$$

$$(\ln x) = \frac{1}{x}$$

4.3 Solve differential equation

Solve $y' = y$

Method 1: educated guessing

Let $y = e^x$

Check: $y' = e^x$

$y = e^x = y'$ ✓

Let $y = ce^x$

$y' = ce^x$

$\Rightarrow y = y'$

Soln: $y = ce^x$

EX: Solve $y' = ky$

Guess $y = Ce^{kx}$

Check: $y' = Ce^{kx} \cdot k$

$= Cke^{kx}$

$$\frac{dy}{dx} = ky$$

$$Cke^{kx} = k\left(Ce^{kx}\right) \checkmark$$

$$Cke^{kx} = kCe^{kx} \checkmark$$

Method 2: Solve $y' = ky$

derivative

$$\frac{y'}{y} = k$$

anti-derivative

$$\ln y = kx + C_1$$

$$e^{\ln y} = e^{kx + C_1} = e^{kx}\underbrace{e^{C_1}}$$

$$\boxed{y = Ce^{kx}}$$

Let $C = e^{C_1}$

Thm 8: Suppose c, K constants

$$\boxed{\frac{dy}{dx} = ky \iff y = ce^{kx}}$$

family of sol'n depending on C

$$\underbrace{\frac{dy}{dx}}_{\text{the rate of change of } y \text{ with respect to } x} \underbrace{=}_{\text{is}} \underbrace{ky}_{\text{proportional to } y}$$

$y = k e^{cx}$ $\boxed{\text{not a sol'n}}$

$y' = k c e^{cx}$

$y' = k y$

$k c e^{cx} \neq k \cdot k e^{cx}$

Better guess $y = C e^{kx}$

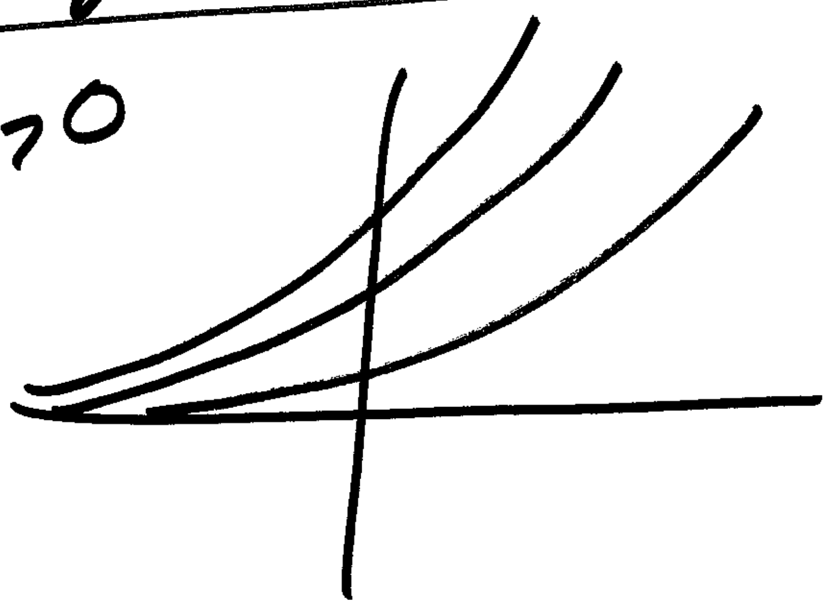
Thm 8 $\Rightarrow$

$$y' = k y \iff y = ce^{kx}$$

If rate of change of y wrt x is proportional to y $\iff$ exponential growth

$K > 0$

$K > 0$



EX: Rate of growth of money is a CD

EX: Rate of growth of bacteria in a closed petri dish under conditions of uninhibited growth

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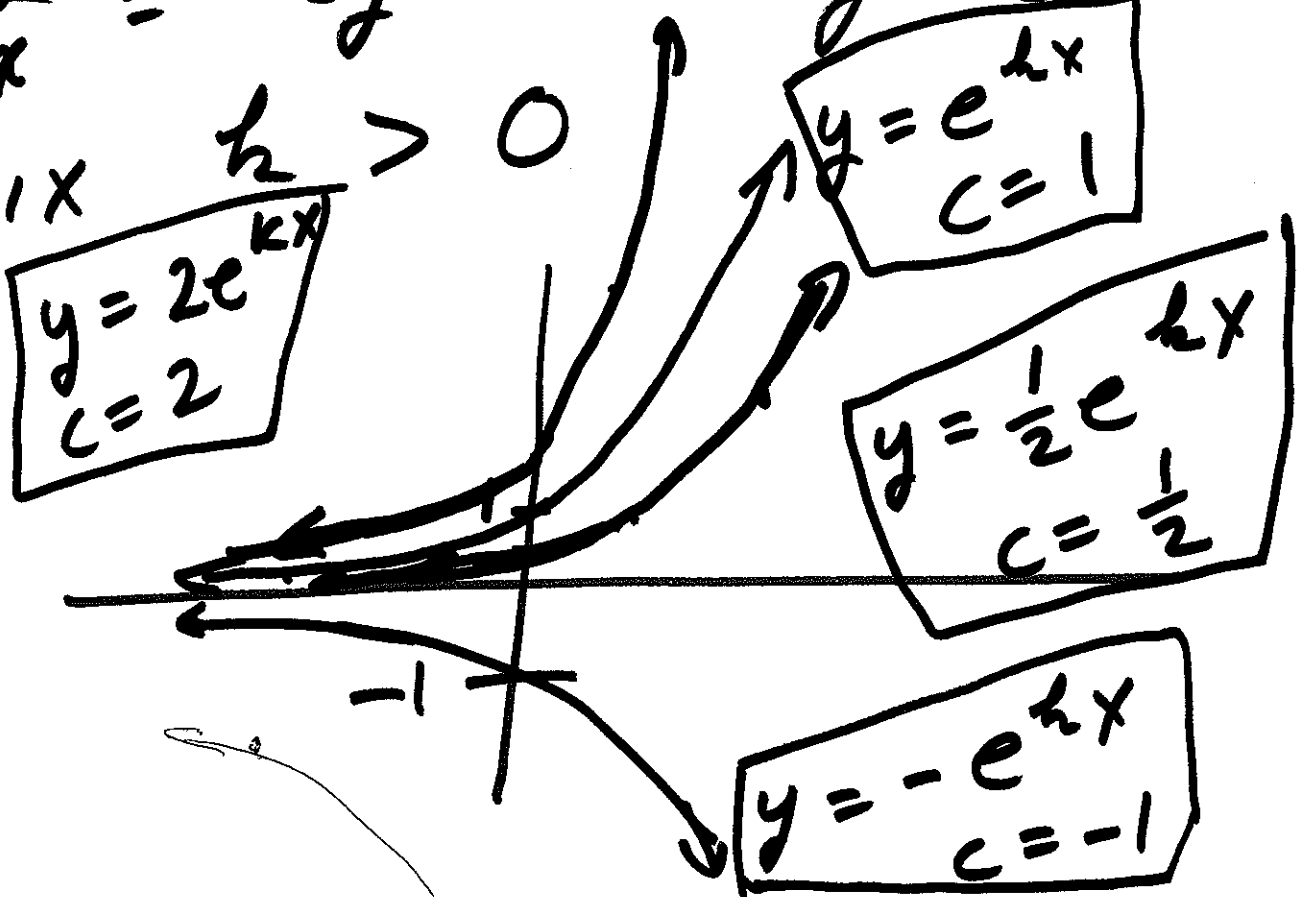
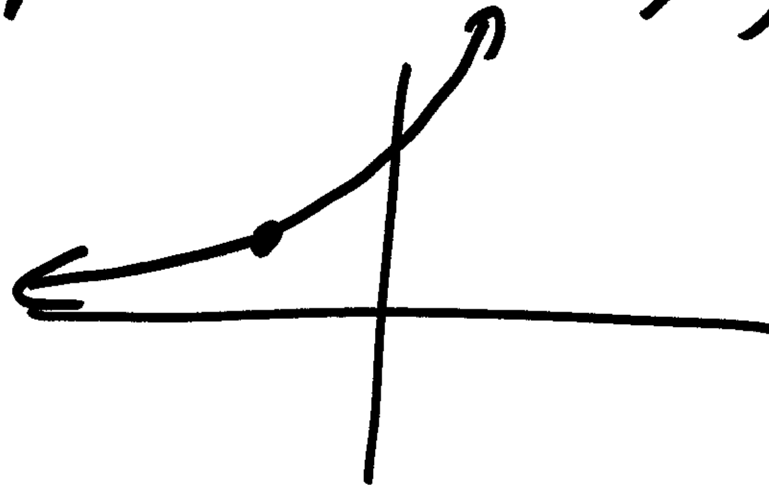
$$\frac{dy}{dx} = ky \iff y = ce^{kx}$$

Fix $k > 0$



x	y
0	c

To specify which $y = ce^{kx}$
Need to choose a point (x_0, y_0)



Initial value problem

Solve $\frac{dy}{dx} = ky$ where $y(x_0) = y_0$

(ie (x_0, y_0) lies on graph of the sol'n)

There exists a unique sol'n to this I.V.P.