5.1) Anti-derivative

Derivative

$(t^2)' \rightarrow 2t$

$(t^2+5)' \rightarrow 2t$

$(t^2+C)' \rightarrow 2t$

Ch 5 : anti-derivative

derivative

$$\int 2t\,dt = t^2 + C$$

anti-derivative

derivative

$$\int t^3\,dt = \frac{t^4}{4} + C$$

anti-derivative

If $f'(t) = t^3$

derivative

anti derivative

$$f(t) = \frac{t^4}{4} + C$$

Initial Value Problem (IVP)

$f'(t) = t^3$, $f(0) = 2$

$$f(t) = \frac{t^4}{4} + C$$

$f(0) = 2$: $2 = \frac{0^4}{4} + C$

$\Rightarrow C = 2$

$$\boxed{f(t) = \frac{t^4}{4} + 2}$$

Suppose

Velocity = $v(t) = 4$ mph $\Rightarrow$ distance traveled $= s(t)$

at time t in hours

$$s(t) = 4t$$

if $s(0) = 0$

Suppose $s(0) = 3$

$$s'(t) = v(t) = 4$$

$$s(t) = 4t + 3$$

If $s'(t) = v(t) = 4$

$s(0) = C$

$\Rightarrow s(t) = 4t + C$

derivative $\leftarrow$ $s(t) \rightarrow 4t + C$

anti-derivative

IVP: Suppose $s'(t) = v(t) = 4$

and $\boxed{s(1) = +2}$

$$s'(t) = 4$$

$$s(t) = 4t + C$$

$s(1) = 2$: $\quad 2 = 4(1) + C$

$$\Rightarrow \quad C = 2 - 4 = -2$$

$$\boxed{s(t) = 4t - 2}$$

Check: $s'(t) = 4$ ✓

$s(1) = 4(1) - 2 = 2$ ✓

$$s(3) = 4(3) - 2$$

$$= 12 - 2 = \boxed{10}$$

Suppose $s'(t) = v(t) =$

$v(t) = t^3 + 3t^2 + 4t^0$, $s(0) = 5$

$\Rightarrow$ (take anti-derivative)

$$s(t) = \frac{t^4}{4} + \frac{\cancel{3}t^3}{\cancel{3}} + 4t + C$$

$s(0) = 5$: $5 = 0 + 0 + 0 + C$

$\Rightarrow C = 5$

$$\boxed{s(t) = \frac{t^4}{4} + t^3 + 4t + 5}$$

Suppose $v(t) = e^t$, $s(0) = 0$

$s'(t) = v(t) = e^t$

$\Rightarrow$

$$s(t) = e^t + C$$

$s(0) = 0$: $0 = e^0 + C \Rightarrow C = -1$

Check $s'(t) = e^t$

$$\boxed{s(t) = e^t - 1}$$

Suppose

$$f'(x) = \sin(x), \quad f(\pi) = 2$$

$\Rightarrow$

$$f(x) = -\cos(x) + C$$

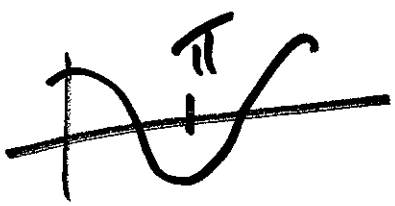
Check: $f'(x) = -[-\sin x] + 0$

$f(\pi) = 2$:

$$2 = -\cos(\pi) + C$$

$$2 = -(-1) + C$$

$$2 = 1 + C \Rightarrow C = 1$$



$$\boxed{f(x) = -\cos(x) + 1}$$

Suppose

$$g'(t) = e^{2t} - 6t^2 + 5t^8 + 3$$

$$g(0) = 0$$

$$\Rightarrow g(t) = \frac{e^{2t}}{2} - 2t^3 + \frac{5t^9}{9} + 3t + C$$

check:

$$g'(t) = \frac{2e^{2t}}{2} - 6t^2 + 5t^8 + 3$$

$g(0) = 0$:

$$0 = \frac{e^0}{2} - 0 + 0 + 0 + C$$

$$0 = \frac{1}{2} + C \Rightarrow C = -\frac{1}{2}$$

$$g(t) = \frac{e^{2t}}{2} - 2t^3 + \frac{5t^9}{9} + 3t - \frac{1}{2}$$