Think before you simplify
Simplification is not always helpful

[8] 5.) Find the following limit: $lim_{x \to 0^+} \frac{e^{\frac{1}{x}}}{x}$

A) $-\infty$ B) $-e$ C) $\frac{-e}{2}$ D) -1 E) 0 F) 1 G) $\frac{e}{2}$ H) e I) 3 J) ∞

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$$

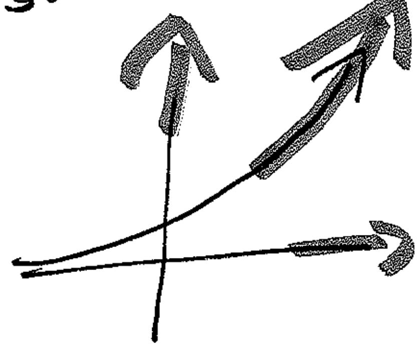
if f continuous

$$\lim_{x \to 0^+} \frac{e^{1/x}}{x} = \frac{+\text{large}}{+\text{small}} = +\infty$$

$$x \to 0^+ \implies \frac{1}{x} \to +\infty$$

$$\frac{1}{10^{-30}} = +10^{30}$$

$$\implies e^{+\text{large}} \longrightarrow +\infty$$



$$\boxed{\implies \lim_{x \to 0^+} e^{1/x} = +\infty}$$

Simplify first

[8] 9.) If $f(x) = ln(\frac{\sqrt{2x+3}}{e^{3x}})$, then $f'(-1) =$

A) $-\frac{5}{2}$ B) -2 C) -1 D) $-\frac{1}{2}$ E) 0 F) $\frac{1}{2}$ G) 1 H) 2 I) $\frac{5}{2}$ J) e^3

$$\ln\left(\frac{\sqrt{2x+3}}{e^{3x}}\right) = \ln\left(\sqrt{2x+3}\right) - \ln e^{3x}$$

$$\left[\ln(2x+3)^{1/2} - \ln e^{3x}\right]'$$

$$= \left[\frac{1}{2}\ln(2x+3) - 3x\right]'$$

$$= \frac{1}{\cancel{2}}\,\frac{\cancel{2}}{2x+3} - 3$$

$$= \frac{1}{2x+3} - 3$$

$$f(-1) = \frac{1}{2(-1)+3} - 3 = 1 - 3 = -2$$

[8] 9.) If $f(x) = ln(\frac{\sqrt{2x+3}}{e^{3x}})$, then $f'(-1) =$

A) $-\frac{5}{2}$ B) -2 C) -1 D) $-\frac{1}{2}$ E) 0 F) $\frac{1}{2}$ G) 1 H) 2 I) $\frac{5}{2}$ J) e^3

$$\left[\ln\left(\frac{\sqrt{2x+3}}{e^{3x}}\right)\right]'$$

$$= \frac{1}{\frac{\sqrt{2x+3}}{e^{3x}}} \cdot \left(\frac{\sqrt{2x+3}}{e^{3x}}\right)'$$

$$= \frac{e^{3x}}{\sqrt{2x+3}} \cdot \frac{e^{3x} \cdot (\sqrt{2x+3})' - (\sqrt{2x+3}) \cdot (e^{3x})'}{(e^{3x})^2}$$

very long method

$$= \frac{e^{3x}}{\cancel{\sqrt{2x+3}}} \cdot \frac{e^{3x}\left(\cancel{\frac{1}{\sqrt{2x+3}}}\right) - (\cancel{\sqrt{2x+3}}) \cdot (3e^{3x})}{e^{6x}}$$

1 1 1

(1 − 3)

$$= \frac{e^{3x}}{\cancel{\sqrt{2x+3}}} \cdot \frac{e^{3x}(\cancel{\ldots})}{e^{6x}}$$

$$= \frac{\left(\frac{1}{\sqrt{2x+3}} - 3\sqrt{2x+3}\right)}{\sqrt{2x+3}} \cdot \frac{e^{3x} \cdot e^{3x}}{e^{6x}}$$

$$= \left(\frac{\frac{1}{\sqrt{2x+3}} - 3\sqrt{2x+3}}{\sqrt{2x+3}}\right)$$

$$f'(-1) = \frac{\frac{1}{\sqrt{-2+3}} - 3\sqrt{-2+3}}{\sqrt{-2+3}} = \frac{1-3}{1} = \frac{-2}{1} = -2$$

Do correct simplifications

$$2 = \ln(e^2) = \ln(e) + \ln(e) = 2\ln e = 2$$

$$0 = \ln(1 \cdot 1) \overset{\checkmark}{=} \ln 1 + \ln 1 = 0$$

$$\ln(ab) = \ln a + \ln b$$

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

$$\ln(a^r) = r \ln a$$

$$\ln(1) = 0$$

$$\ln(a+b) = \ln(a+b)$$

$$\ln(1+1) = \ln 2$$

$$\int_{-1}^{1} \frac{2e^{3x}}{5+e^{3x}}\,dx$$

Let $u = 5+e^{3x}$

$$\frac{du}{3} = \frac{(\cancel{3}e^{3x})\,dx}{\cancel{3}}$$

$$\int_{-1}^{1} \frac{2e^{3x}\,dx}{5+e^{3x}} = \int \frac{2\,du}{3u}$$

$\cancel{= \ln|u|\Big|_{-1}^{1}}$ ~~except~~

$x = 1 \implies u = 5+e^{3}$

$x = -1 \implies u = 5+e^{-3}$

Method 1

$$\int_{-1}^{1} \frac{2e^{3x}\,dx}{5+e^{3x}} = \int_{5+e^{-3}}^{5+e^{3}} \frac{2\,du}{3u}$$

$$= \frac{2}{3}\ln|u|\Big|_{5+e^{-3}}^{5+e^{3}}$$

$$= \frac{2}{3}\ln|5+e^{3}| - \ln|5+e^{-3}|$$

$$= \frac{2}{3}\ln\left|\frac{5+e^{3}}{5+e^{-3}}\right|$$

Method 2

$$\int_{-1}^{1} \frac{2e^{3x}\,dx}{5+e^{3x}} = \frac{2}{3}\int \frac{du}{u}$$

$$= \frac{2}{3} \ln|u| \Big|$$

$$= \frac{2}{3} \ln|5+e^{3x}| \Big|_{-1}^{1}$$

$$= \frac{2}{3} \ln\left|\frac{5+e^{3}}{5+e^{-3}}\right|$$

$$\int_{-1}^{1} 2\left[\cos\left(5+e^{3x}\right)\right]\left(e^{3x}\,dx\right)$$

Let $u = 5 + e^{3x}$

$$du = 3\left(e^{3x}\,dx\right)$$

$$\frac{du}{3} = \frac{\cancel{3}\left(e^{3x}\,dx\right)}{\cancel{3}}$$

$$\int \frac{2\cos(u)\,du}{3}$$

$$= \frac{2}{3}\sin u$$

$$= \frac{2}{3}\sin\left(5+e^{3x}\right)\Big|_{-1}^{1}$$

$$= \frac{2}{3}\sin\left(5+e^{3}\right) - \frac{2}{3}\sin\left(5+e^{-3}\right)$$

$x = 1 \Rightarrow u = 5 + e^{3}$

$x = -1 \Rightarrow u = 5 + e^{-3}$

$$\int_{-1}^{1} 2 \cos(5 + e^{3x})(e^{3x} dx)$$

$$= \frac{2}{3} \int_{5+e^{-3}}^{5+e^{+3}} \cos u \, du$$

$=$

$$\int_{-1}^{1} 2e^{3x}\sqrt{5+e^{3x}}\,dx$$

$$\int_{-1}^{1} 2\underline{e^{3x}}\, e^{(5+e^{3x})}\, \underline{dx}$$

Find the average value of $f(x) = \frac{2e^{3x}\,dx}{5+e^{3x}}$

over $[-1, 1]$

$$\underbrace{\frac{1}{1-(-1)}}_{\text{divide by length of interval}} \underbrace{\int_{-1}^{1} \frac{2e^{3x}\,dx}{5+e^{3x}}}_{\text{Add up all \#'s}}$$

to get average

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{3}\ln\left|\frac{5+e^{3}}{5+e^{-3}}\right|\right]$$

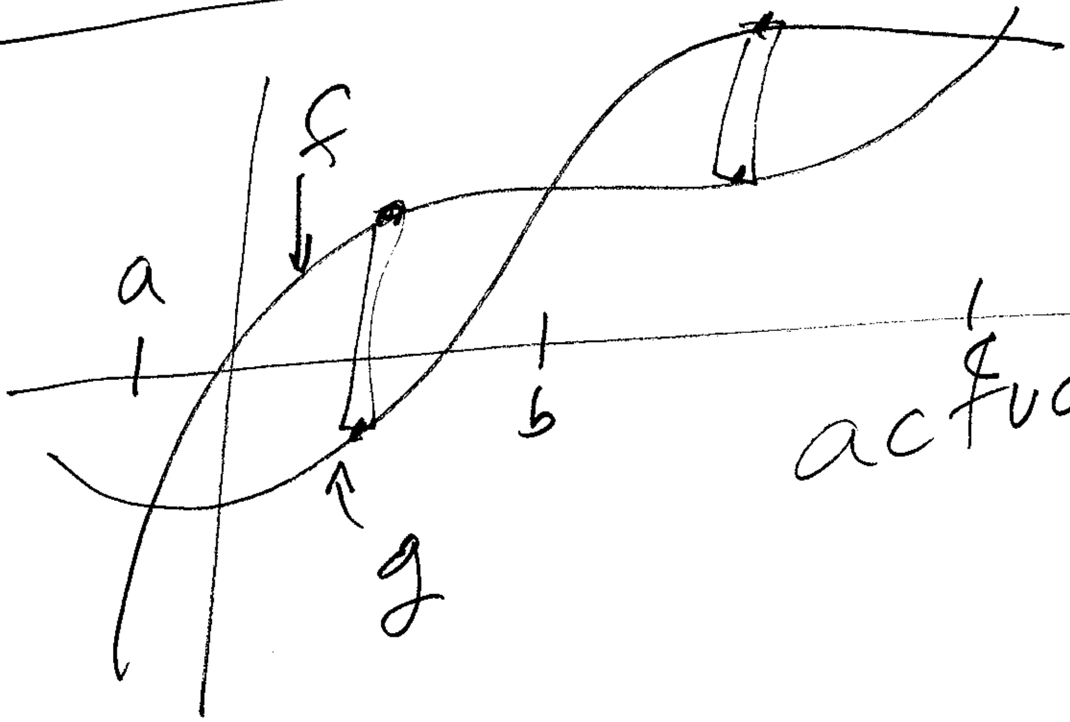
Find the average ~~of~~ value of $f(x) = 2e^{3x}\cos(5+e^{3x})$ over $[-1, 1]$

$$\underbrace{\underbrace{\frac{1}{1-(-1)}}_{\substack{\text{divide}\\\text{by length}\\\text{of interal}}} \underbrace{\int_{-1}^{1} 2e^{3x}\cos(5+e^{3x})\,dx}_{\substack{\text{Add of all}\\\#\text{'s}\\\text{(sort of}}}}_{\text{gives average}}$$

$$= \frac{1}{3}\left[\sin(5+e^{3}) - \sin(5+e^{-3})\right]$$

$$\int_a^b f(x)\,dx = \text{net area}$$

VS



actual area

$$\int_a^b (f-g)\,dx + \int_b^c (g-f)\,dx$$

5.9) post ex: definite integral

$$\int_0^b e^{-x}\,dx = -e^{-x}\Big|_0^b$$

$$= -e^{-b} - (-e^{-0})$$

$$= 1 - e^{-b}$$

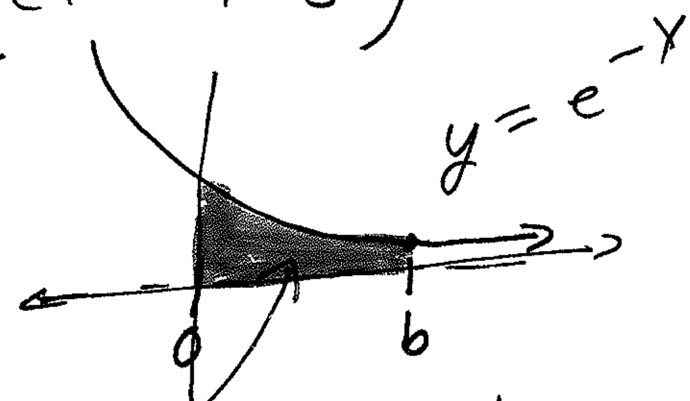
= area btwn

$y = 0$ (x-axis)

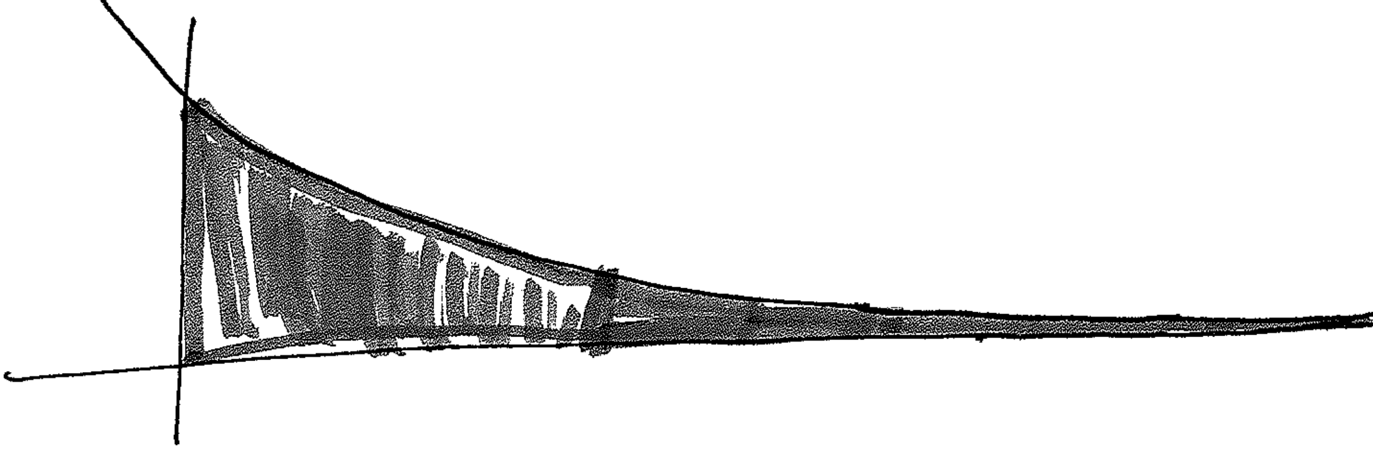
$y = e^{-x}$

$x = 0$

$x = b$



Note this area $= 1 - e^{-b} < 1$

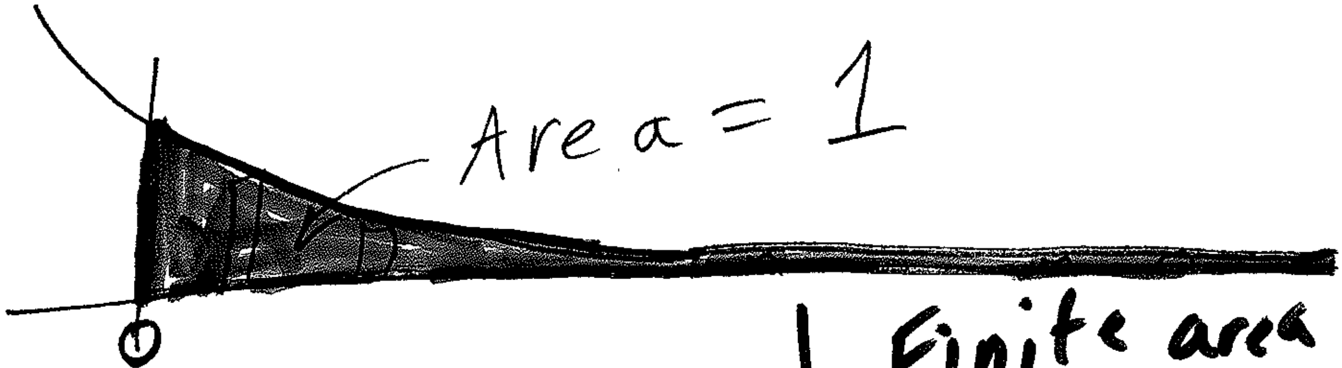


Find area betwee

$y = 0$ (x-axis

$y = e^{-x}$

and $x \geq 0$



$= \int_0^\infty e^{-x}\,dx$

$= \lim_{b \to \infty} \int_0^b e^{-x}\,dx$

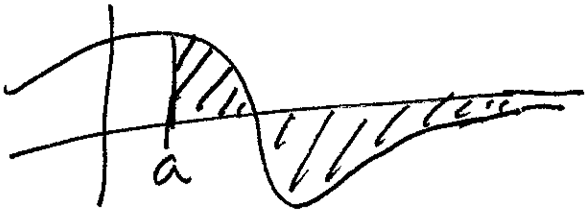
Finite area makes sense since $\int_0^b e^{-x} < 1$

$= \lim_{b \to \infty} \left[1 - e^{-b}\right] = 1$

0

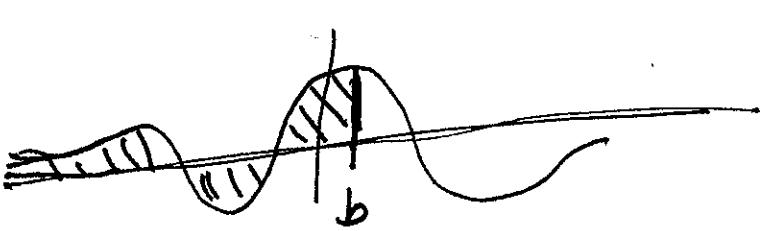
$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$$

= net area btwn $y = f(x)$
$y = 0$ (x-axis)
where $x \geq a$



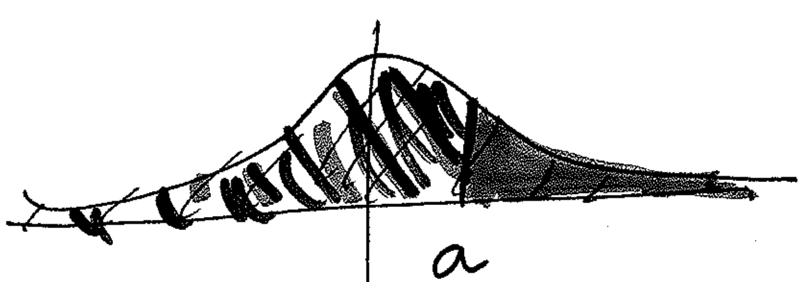
$$\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty} \int_a^b f(x)\,dx$$

= net area btwn $y = f(x)$
$y = 0$ (x-axis)
where $x \leq b$



$$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^a f(x)\,dx + \int_a^\infty f(x)\,dx$$

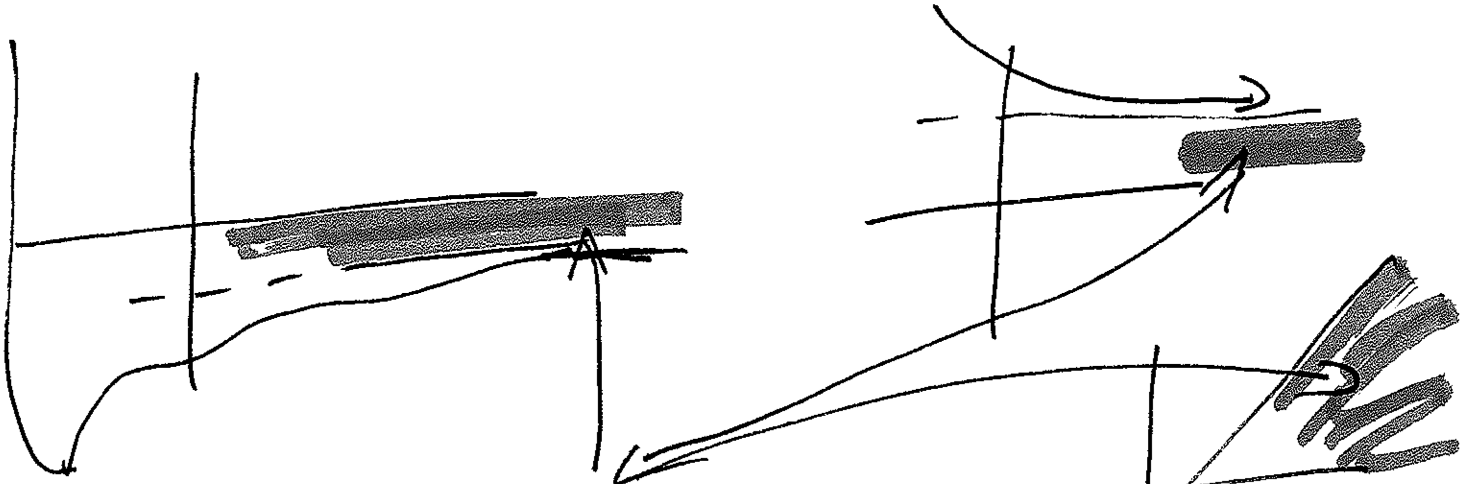
~~net area~~



Definitions of <u>Improper</u> Integral

$$\lim_{x \to +\infty} f(x) \neq 0 \Rightarrow \int_a^{\infty} f(x)\,dx \text{ DIV}$$

DNE



area

NOT finite

$$\lim_{x \to -\infty} f(x) \neq 0 \Rightarrow \int_{-\infty}^{b} f(x)\,dx$$

DIV

(DNE)

$$\lim_{x \to +\infty} f(x) = 0 \Rightarrow \int_a^{+\infty} f(x)\,dx$$

$= ?$

$-\infty \leftarrow$ similar