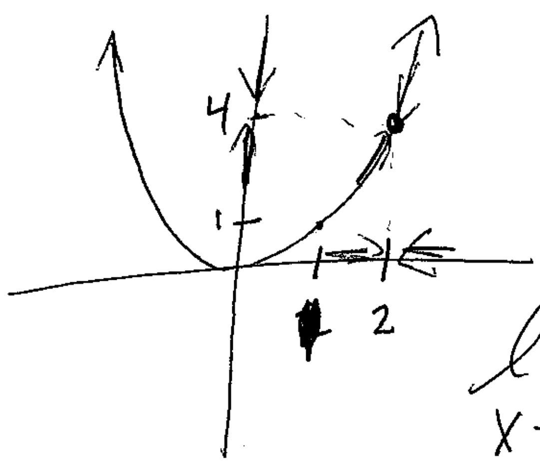
2.1, 2.2

Defn: $\lim_{x \to a} f(x) = L$ if

x close to a (except possibly at a)

$\Rightarrow f(x)$ is close to L

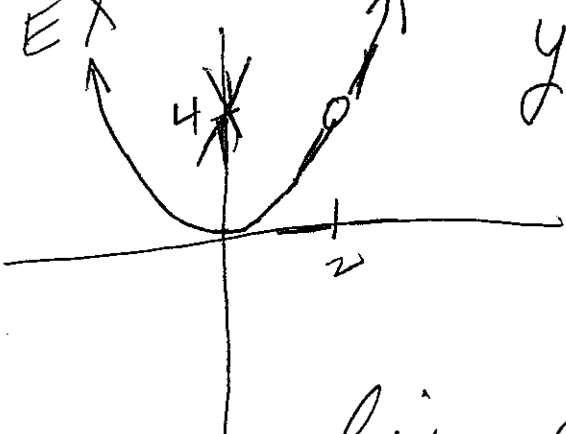
EX



$y = x^2$

$\lim_{x \to 2} x^2 = 4$

EX



$y = g(x) = x^2 \cdot \underbrace{\frac{(x-2)}{(x-2)}}_{\text{multiplied by a form of 1}} = \frac{x^3 - 2x^2}{x-2}$

$\lim_{x \to 2} g(x) = 4$

1

$$\lim_{x \to 2} \frac{x^3 - 2x^2}{x-2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x^2)}{\cancel{x-2}}$$

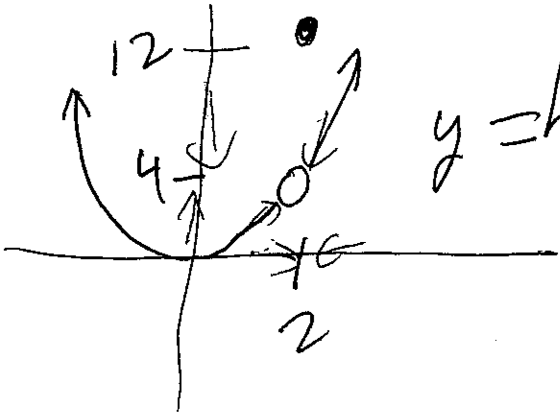
$$\text{"}\frac{0}{0}\text{"}$$ ← simplify
ie factor usually

$$= \lim_{x \to 2} x^2 = 4$$

since we don't care what happens at 2 (only near 2)

$$x^2 \neq \frac{(x-2)x^2}{x-2}$$

when $x = 2$



$$y = h(x) = \begin{cases} x^2, & x \neq 2 \\ 12, & x = 2 \end{cases}$$

$$\lim_{x \to 2} h(x) = 4$$

2.1	~ 4
2.001	~ 4

$$\lim_{x \to 2} \frac{x+1}{x-2} = \boxed{DNE}$$

$$\text{"}\frac{3}{0}\text{"}$$

Explanation:

closer to 2 ↓

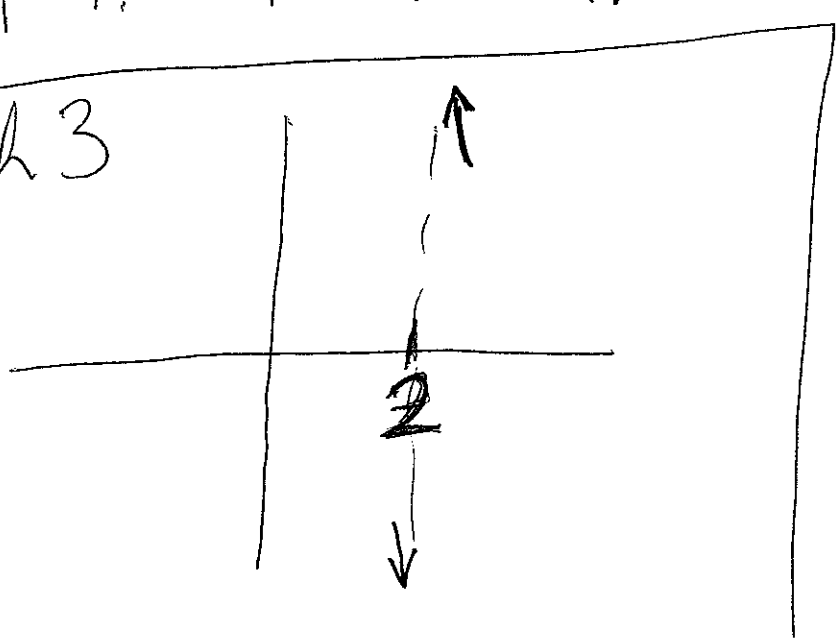
x	$(x+1)/(x-2)$
2.1	$3.1/0.1 = \frac{3.1}{1/10} = \frac{3.1}{10^{-1}} = (3.1)\times 10^1 = 31$
2.01	$3.01/0.01 = \frac{3.01}{10^{-2}} = 3.01\times 10^2 = 301$
2.00001	$3.00001/0.00001 = 3.00001\times 10^5 = 300{,}001$
1.999	$2.999/-0.001 = -2999$
1.9	$2.9/-0.1 = -2.9\times 10 = -29$

closer to 2 ↑

getting large $\to \infty$

get large but in negative direction $\to -\infty$

Ch 3



2.1 One-sided limits

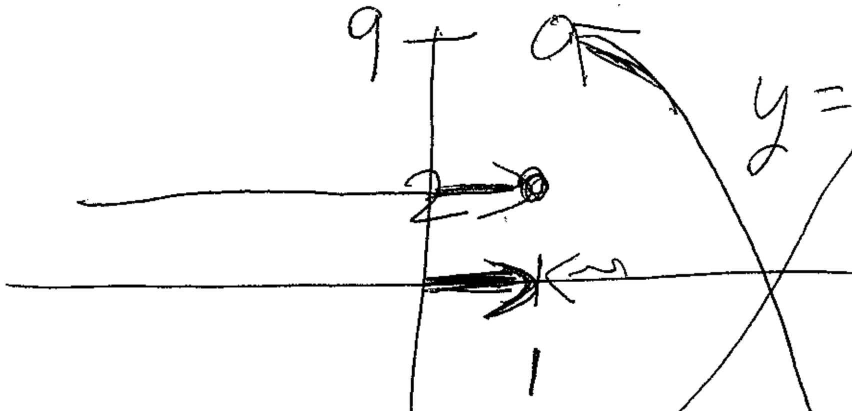
Defn $\lim\limits_{x \to a^-} f(x) = L$ if

x close to a, $x < a \Rightarrow f(x)$ close to L

Defn $\lim\limits_{x \to a^+} f(x) = L$ if

x close to a, $x > a \Rightarrow f(x)$ close to L

Ex



$$y = f(x) = \begin{cases} 2, & x \le 1 \\ 10 - x^2, & x > 1 \end{cases}$$

$\lim\limits_{x \to 1^-} f(x) = 2$ $\qquad$ $\lim\limits_{x \to 1^+} f(x) = 9$